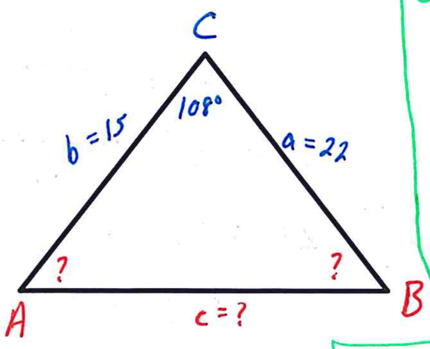
$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Emphasis on Solving Triangles Using the Law of Cosines

CHOOSE THE EQUATION BASED ON ANGLE GIVEN

Solve the following triangles given the provided information:

1. ∠C = 108°, a = 22, & b = 15



① $c^2 = 22^2 + 15^2 - 2(22)(15)\cos 108$

$c^2 = 484 + 225 - 660\cos 108$

$c^2 = 709 - 660\cos 108$

$\sqrt{c^2} \approx \sqrt{912.9512163}$

$c \approx 30.2150826$

$\boxed{c \approx 30.2}$

② $\underbrace{\dfrac{\sin A}{22} = \dfrac{\sin B}{15} = \dfrac{\sin 108}{30.2}}$

$\dfrac{\cancel{30.2}\sin B}{\cancel{30.2}} = \dfrac{15\sin 108}{30.2}$

$\cancel{\sin^{-1}}(\cancel{\sin} B) = \sin^{-1}\left(\dfrac{15\sin 108}{30.2}\right)$

$\angle B = \sin^{-1}\left(\dfrac{15\sin 108}{30.2}\right)$

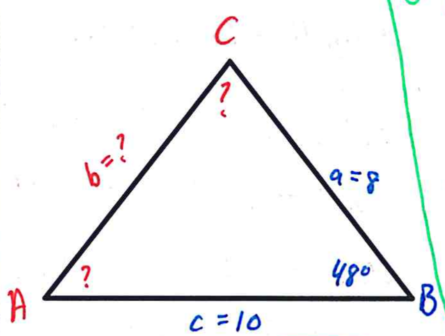
$\angle B \approx 28.1888379$

$\boxed{\angle B \approx 28.2°}$

③ $\angle A \approx 180 - 108 - 28.2$

$\boxed{\angle A \approx 43.8°}$

2. ∠B = 48°, a = 8, & c = 10



① $b^2 = 8^2 + 10^2 - 2(8)(10)\cos 48$

$b^2 = 64 + 100 - 160\cos 48$

$b^2 = 164 - 160\cos 48$

$\sqrt{b^2} \approx \sqrt{56.93910298}$

$b \approx 7.545800354$

$\boxed{b \approx 7.5}$

② $\underbrace{\dfrac{\sin A}{8} = \dfrac{\sin 48}{7.5}} = \dfrac{\sin C}{10}$

$\dfrac{\cancel{7.5}\sin A}{\cancel{7.5}} = \dfrac{8\sin 48}{7.5}$

$\sin^{-1}(\sin A) = \sin^{-1}\left(\dfrac{8\sin 48}{7.5}\right)$

$\angle A = \sin^{-1}\left(\dfrac{8\sin 48}{7.5}\right)$

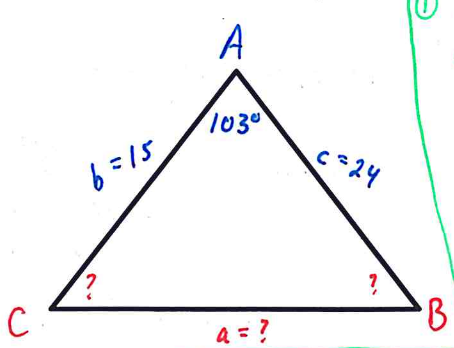
$\angle A \approx 52.43740612$

$\boxed{\angle A \approx 52.4°}$

③ $\angle C \approx 180 - 48 - 52.4$

$\boxed{\angle C \approx 79.6°}$

3. ∠A = 103°, b = 15, & c = 24



① $a^2 = 15^2 + 24^2 - 2(15)(24)\cos 103$

$a^2 = 225 + 576 - 720\cos 103$

$a^2 = 801 - 720\cos 103$

$\sqrt{a^2} \approx \sqrt{962.9647591}$

$a \approx 31.03167348$

$\boxed{a \approx 31}$

② $\underbrace{\dfrac{\sin 103}{31} = \dfrac{\sin B}{15}} = \dfrac{\sin C}{24}$

$\dfrac{\cancel{31}\sin B}{\cancel{31}} = \dfrac{15\sin 103}{31}$

$\cancel{\sin^{-1}}(\cancel{\sin} B) = \sin^{-1}\left(\dfrac{15\sin 103}{31}\right)$

$\angle B = \sin^{-1}\left(\dfrac{15\sin 103}{31}\right)$

$\angle B \approx 28.12971991$

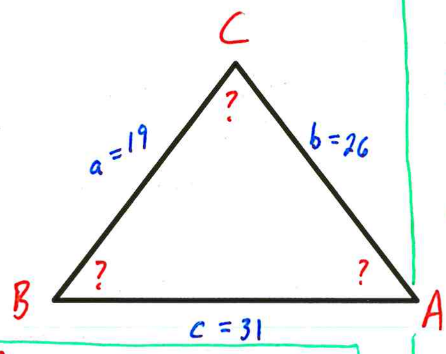
$\boxed{\angle B \approx 28.1°}$

③ $\angle C \approx 180 - 103 - 28.1$

$\boxed{\angle C \approx 48.9°}$

I'm using $a^2 = b^2 + c^2 - 2bc \cdot \cos A$ for this side.

4. a = 19, b = 26, & c = 31



① $19^2 = 26^2 + 31^2 - 2(26)(31)\cos A$

$361 = 676 + 961 - 1612\cos A$

$361 = 1637 - 1612\cos A$

$-1637 \quad -1637$

$\frac{-1276}{-1612} = \frac{-1612\cos A}{-1612}$

$0.7915632754 \approx \cos A$

$\cos^{-1}(0.7915632754) \approx \angle A$

$37.66815707 \approx \angle A$

$\boxed{37.7° \approx \angle A}$

② $\underbrace{\frac{\sin 37.7}{19} = \frac{\sin B}{26}} = \frac{\sin C}{31}$

$\frac{19 \sin B}{19} = \frac{26 \sin 37.7}{19}$

$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{26 \sin 37.7}{19}\right)$

$\angle B = \sin^{-1}\left(\frac{26 \sin 37.7}{19}\right)$

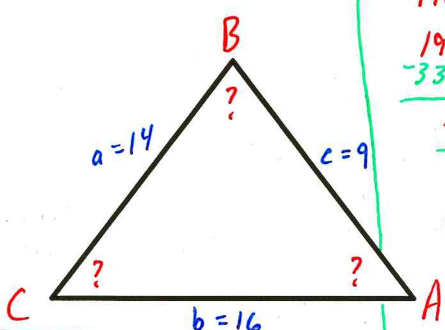
$\angle B \approx 56.80650512$

$\boxed{\angle B \approx 56.8°}$

③ $\angle C \approx 180 - 37.7 - 56.8$

$\boxed{\angle C \approx 85.5°}$

5. a = 14, b = 16, c = 9



① $14^2 = 16^2 + 9^2 - 2(16)(9)\cos A$

$196 = 256 + 81 - 288\cos A$

$196 = 337 - 288\cos A$

$-337 \quad -337$

$\frac{-141}{-288} = \frac{-288\cos A}{-288}$

$0.4895833333 \approx \cos A$

$\cos^{-1}(0.4895833333) \approx \angle A$

$60.68680104 \approx \angle A$

$\boxed{60.7° \approx \angle A}$

② $\underbrace{\frac{\sin 60.7}{14} = \frac{\sin B}{16}} = \frac{\sin C}{9}$

$\frac{14 \sin B}{14} = \frac{16 \sin 60.7}{14}$

$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{16 \sin 60.7}{14}\right)$

$\angle B = \sin^{-1}\left(\frac{16 \sin 60.7}{14}\right)$

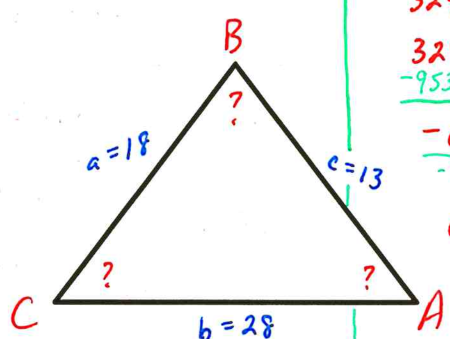
$\angle B \approx 85.30924639$

$\boxed{\angle B \approx 85.3°}$

③ $\angle C \approx 180 - 60.7 - 85.3$

$\boxed{\angle C \approx 34°}$

6. a = 18, b = 28, & c = 13



① $18^2 = 28^2 + 13^2 - 2(28)(13)\cos A$

$324 = 784 + 169 - 728\cos A$

$324 = 953 - 728\cos A$

$-953 \quad -953$

$\frac{-629}{-728} = \frac{-728\cos A}{-728}$

$0.864010989 \approx \cos A$

$\cos^{-1}(0.864010989) \approx \angle A$

$30.23003572 \approx A$

$\boxed{30.2° \approx \angle A}$

② $\underbrace{\frac{\sin 30.2}{18} = \frac{\sin B}{28}} = \frac{\sin C}{13}$

$\frac{18 \sin B}{18} = \frac{28 \sin 30.2}{18}$

$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{28 \sin 30.2}{18}\right)$

$\angle B = \sin^{-1}\left(\frac{28 \sin 30.2}{18}\right)$

$\angle B \approx 51.48778947$

$\boxed{\cancel{\angle B \approx 51.5°}}$

Note: side b is the largest side so $\angle B$ is the largest $\angle$.

$180 - 51.5 = \boxed{128.5° = \angle B}$

③ $\angle C \approx 180 - 30.2 - 128.5$

$\boxed{\angle C \approx 21.3°}$