Geometric Proofs

Topics of emphasis:

1. Proof of triangle congruence by:
    - Side Side Side
    - Side Angle Side
    - Angle Side Angle
    - Angle Angle Side
    - Hypotenuse Leg

2. Proof of triangle similarity by:
    - Angle Angle
    - Side Side Side
    - Side Angle Side

What is a proof?

Definitions:

1. Proof:
   A logical argument showing a statement is true.

2. Two Column Proof:
   A T-chart that has a list of statements on the left side, and the supporting reasons for them on the right side.

3. Paragraph Proof:
   A paragraph that tells how your statements and reasons flow together instead of using a Two Column approach.

Essential Properties

Reflexive Property:

For any real number a,

a = a

Symmetric Property:

For any real numbers a and b,

If a = b, then b = a

Transitive Property:

For any real numbers a, b, and c

If a = b and b = c, then a = c

Making a Two Column Proof

Statements	Reasons
What can you use? Whatever you can provide a reason for. Mainly: - Stating two sides are congruent/similar to each other. - Stating two angles are congruent/similar to each other.	What can you use? Definitions Theorems Postulates Properties Formulas Established Equations

SSS

ASA

Beginning of

Proof of triangle: Congruence!

SAS

HL

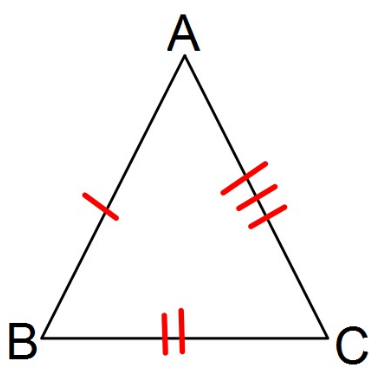
AAS

Side - Side - Side

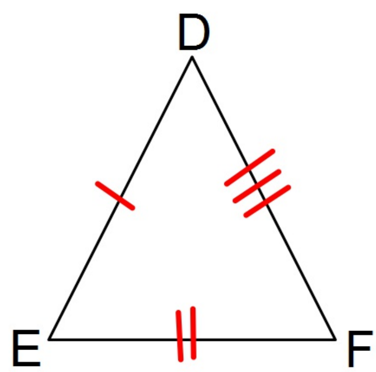
Side - Side - Side (SSS) Congruence Postulate:

IF 3 sides of one triangle are congruent to 3 sides of another triangle,

THEN the two triangles are congruent.



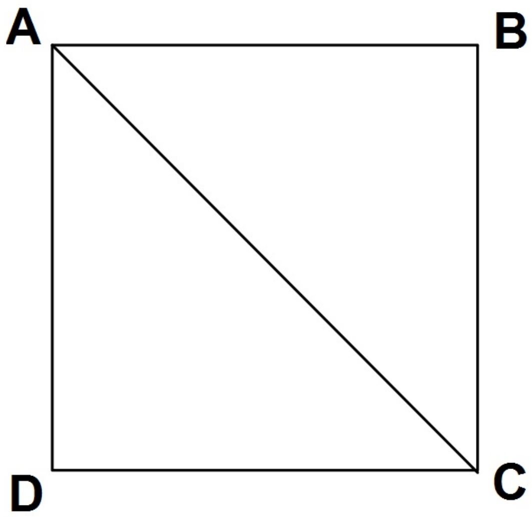
Indicates that:
$\triangle ABC \cong \triangle DEF$



Example

1. **Given:** ABCD is a square

   Prove: △ACD ≅ △CAB

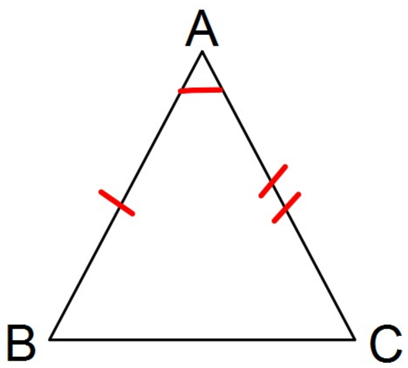


Side - Angle - Side

Side - Angle - Side (SAS) Congruence Postulate:

IF two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle,

THEN the two triangles are congruent.



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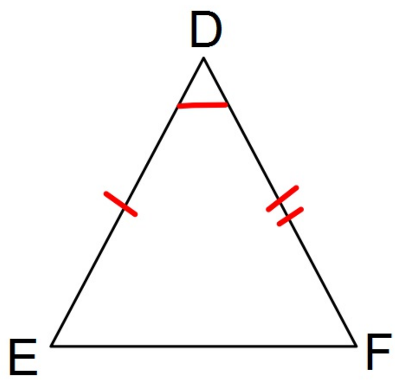
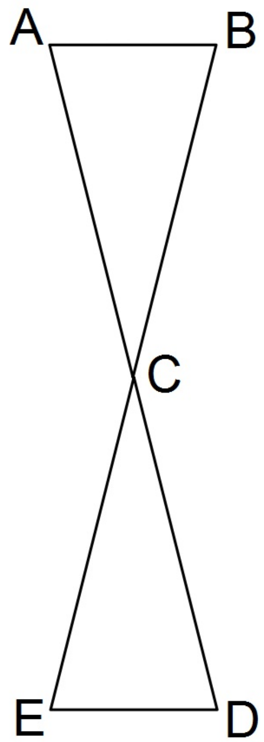


Example

1. **Given:** C is the midpoint of $\overline{AD}$ and $\overline{BE}$

   Prove: $\triangle ACB \cong \triangle DCE$

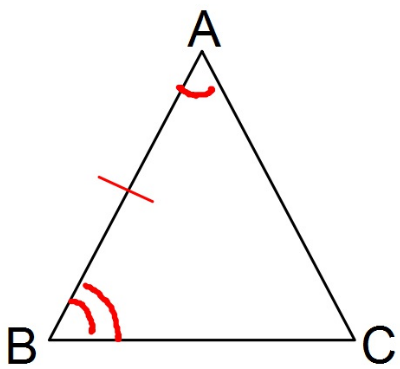


Angle - Side - Angle

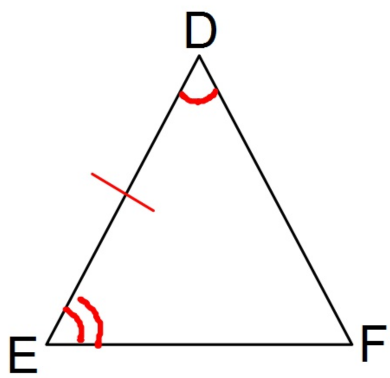
Angle - Side - Angle (ASA) Congruence Postulate:

IF two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle,

THEN the two triangles are congruent.



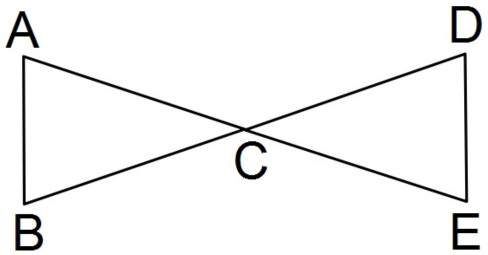
Indicates that:
$\triangle ABC \cong \triangle DEF$



Example

1. **Given:** $\overline{AB} \parallel \overline{DE}$
   C is the midpoint of $\overline{BD}$

   Prove: $\triangle ABC \cong \triangle EDC$

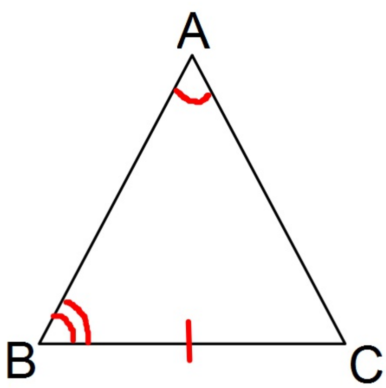


Angle - Angle - Side

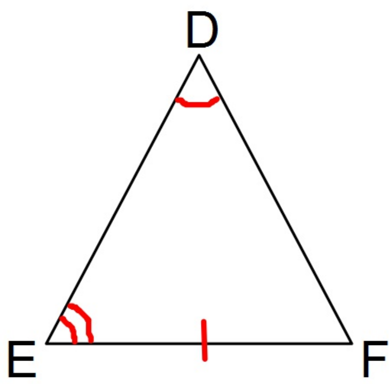
Angle - Angle - Side (AAS) Congruence Postulate:

IF two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle,

THEN the two triangles are congruent.



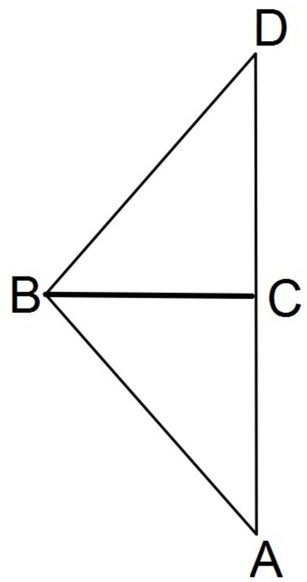
Indicates that:
$\triangle ABC \cong \triangle DEF$



Example

1. **Given:** $\overline{BC} \perp \overline{AD}$
   $\triangle ABD$ is an Isosceles Triangle with base $\overline{AD}$

   Prove: $\triangle ABC \cong \triangle DBC$



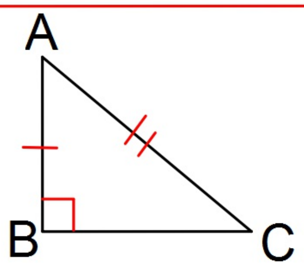
Hypotenuse - Leg

Hypotenuse - Leg (HL) Congruence Theorem:

IF the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and the corresponding leg of a second right triangle,

THEN the two triangles are congruent.

Important Note:
You must show that both triangles are right triangles in order to use HL.



Indicates that:
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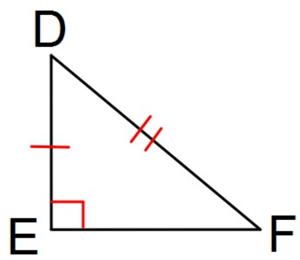
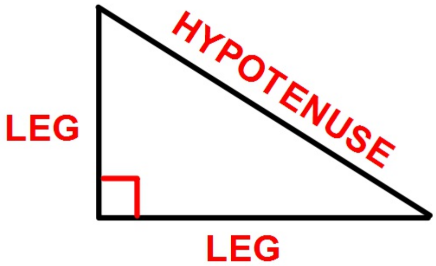


Important Vocabulary for Hypotenuse - Leg

Right Triangles:

In a right triangle, the sides adjacent to the right angle are called the **LEGS**. The side opposite of the right angle is called the **HYPOTENUSE**.



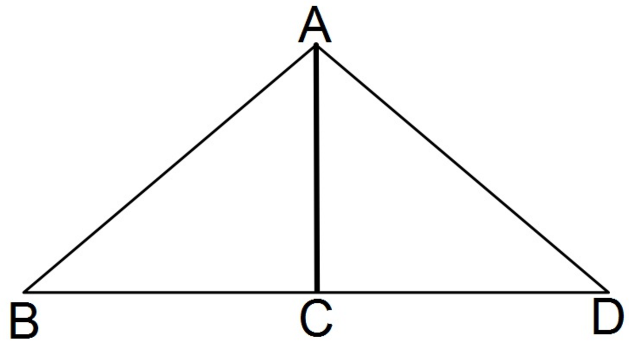
Perpendicular:

Two lines that intersect and form right angles.
Denoted as $\perp$

Example

1. **Given:** $\overline{AB} \cong \overline{AD}$
   $\overline{AC} \perp \overline{BD}$

   Prove: $\triangle ABC \cong \triangle ADC$



Beginning of Proof of triangle Similarity!

AA

SSS

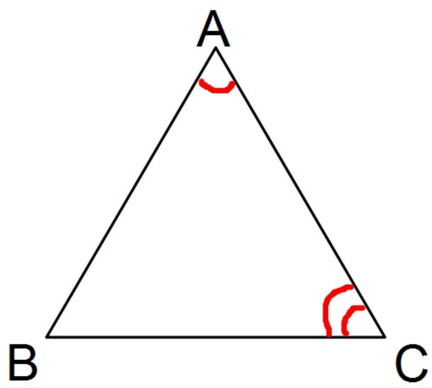
SAS

Angle - Angle

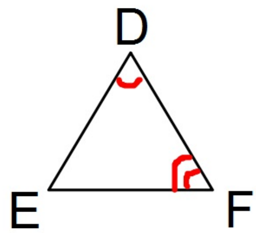
Angle - Angle (AA) Similarity Postulate:

IF two angles of one triangle are congruent to two angles of another triangle,

THEN the two triangles are similar.



Indicates that:
$\triangle ABC \sim \triangle DEF$



Useful Information

Theorems:

1. Triangle Sum Theorem:
   All the angles in a triangle add up to 180^{O}

2. Right Angle Congruence Theorem:
   All right angles are congruent.

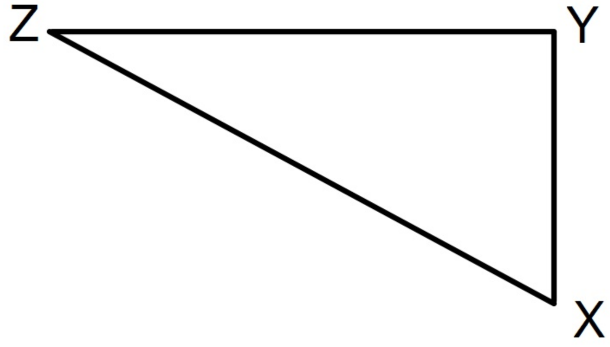
Definition:

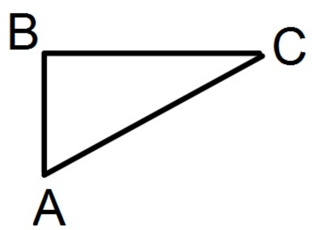
1. Congruence:
   If two things are equal,
   Then they are also congruent.

Example

1. **Given:** $\overline{XY} \perp \overline{YZ}$
   $\overline{AB} \perp \overline{BC}$
   $\angle X = 48^{O}$
   $\angle C = 42^{O}$

   Prove: $\triangle XYZ \sim \triangle ABC$

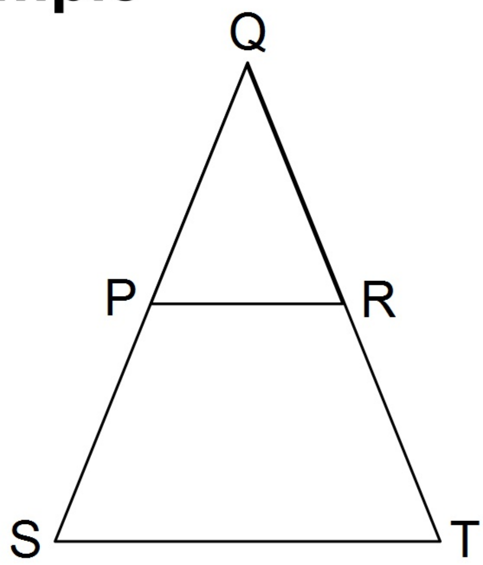




Another Example

2. **Given:** $\overline{PR} \parallel \overline{ST}$

Prove: $\triangle PQR \sim \triangle SQT$

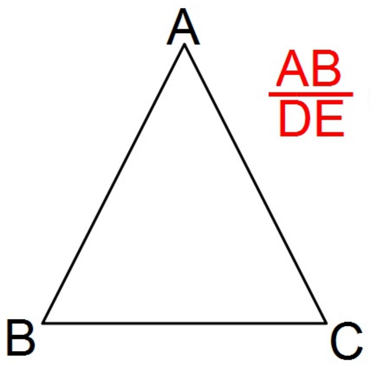


Side - Side - Side

Side - Side - Side (SSS) Similarity Theorem:

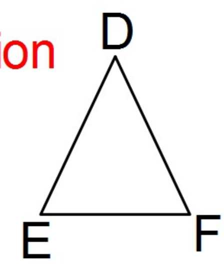
IF the corresponding side lengths of two triangles are proportional,

THEN the two triangles are similar.



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \text{a simplified fraction}$$

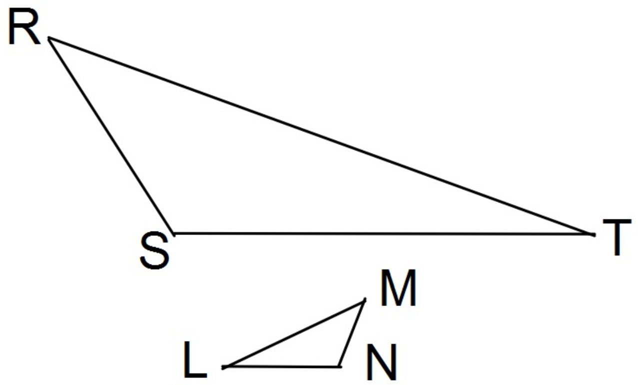
Indicates that:
$\triangle ABC \sim \triangle DEF$



Example

1. **Given:** RS = 9
ST = 15
RT = 27
LM = 18
MN = 6
LN = 10

Prove: $\triangle RST \sim \triangle MNL$



Additional Situation for Side - Side - Side

There will be times when I give you only two of the three sides, however, I also give you that we have right triangles. With this information we can find the third side of each triangle by doing the following:

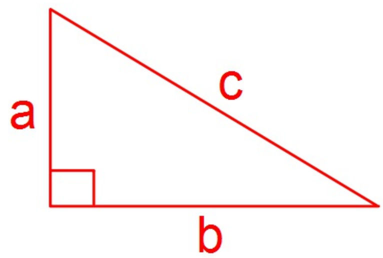
Pythagorean Theorem:

Given a right triangle with legs of length a and b, and the the hypotenuse of length c...

$$a^2 + b^2 = c^2$$

Use this equation to find missing sides,

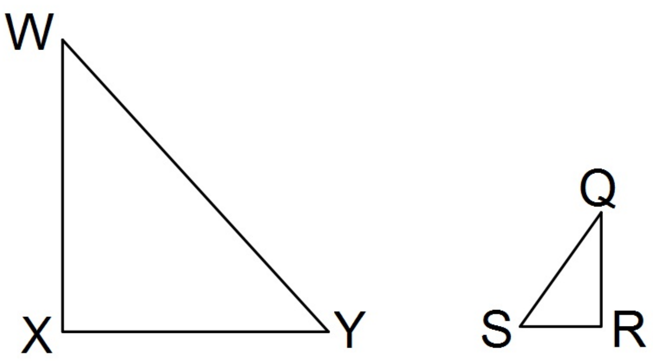
AFTER stating the triangles are right triangles!



Additional Example

2. **Given:** $\overline{WX} \perp \overline{XY}$
$\overline{QR} \perp \overline{RS}$
WX = 20
QR = 5
XY = 48
RS = 12

Prove: $\triangle WXY \sim \triangle QRS$



Side - Angle - Side

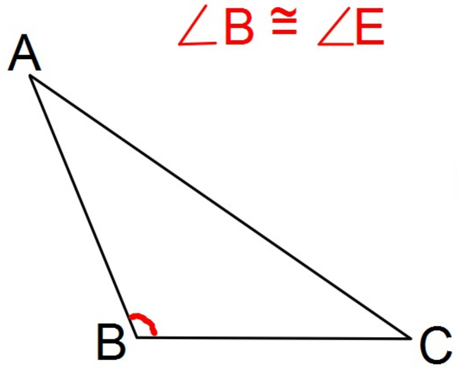
Side - Angle - Side (SAS) Similarity Theorem:

IF an angle of one triangle is congruent to an angle of another triangle AND the lengths of the sides including these angles are proportional,

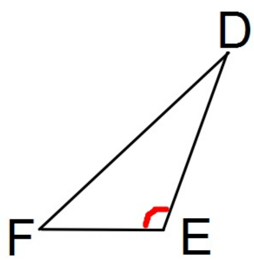
THEN the two triangles are similar.

$\angle B \cong \angle E$

$\frac{AB}{DE} = \frac{BC}{FE}$ = a simplified fraction



Indicates that:
$\triangle ABC \sim \triangle DEF$



Example

1. **Given:** XY = 33
   JH = 44
   XW = 39
   JK = 52
   $\overline{XY} \parallel \overline{JH}$

   Prove: $\triangle XYW \sim \triangle JHK$