Solving Equations Continued

Focus of the Day:

1. Solving Polynomials
    - Rational Root Theorem
    - Descartes Rule of Signs
2. Solving Exponential Equations
3. Solving Logarithmic equations

Solving Polynomials - Part 1

Rational Root Theorem:

The Rational Root Theorem states a constraint on rational solutions of a polynomial equation with integer coefficients.

Procedure:

1. Given an equation: $p_n x^n + p_{n-1} x^{n-1} + \ldots + q = 0$
2. Identify the value of q, this is the constant of the function.
3. Factor out q entirely (Complete the U)
4. Identify the value of p, this is the leading coefficient.
5. Factor out p entirely (Complete the U)
6. Divide each of the numbers in Step 3 by the numbers in Step 5. Simplify each of the fractions.
7. Put a $\pm$ sign in front of each of the results of Step 6. This is your list of ALL possible rational roots of the function given.

Examples

Identify the possible Rational Roots of the given equations:

1. $x^4 - 5x^2 - 36 = 0$

2. $x^4 - x^3 - 5x^2 - 103x + 300 = 0$

Solving Polynomials - Part 2

Descartes Rule of Signs:

A method used to determine the number of REAL zeros of a polynomial function.

Procedure:

1. Ensure that the equation is put into standard form.
2. Find the number of positive REAL zeros:
    A. List all the signs of the coefficients and the constant in order!
    B. Moving left to right, count the number of times the sign changes.
    C. This number, or the number minus 2 until we reach 1 or 0 is equal to the number of positive REAL zeros.
3. Find the number of negative REAL zeros:
    A. Rewrite the equation changing all signs on the terms with an odd powered exponent.
    B. List all the signs of the coefficients and the constant in order!
    C. Moving left to right, count the number of sign changes again.
    D. This number, or the number minus 2 until we reach 1 or 0 is equal to the number of negative REAL zeros.

Examples

Determine the number of real zeros for each of the following:

1. $x^4 - 5x^2 - 36 = 0$

2. $x^4 - 14x^2 + 45 = 0$

More Examples

Determine the number of real zeros for each of the following:

3. $x^3 - 2x^2 - 3x + 6 = 0$

4. $x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$

Solving Polynomials
Finding the Zeros

Procedure:

1. Identify the degree of the function.
   - Keep in mind this is equal to the number of solutions.
2. Use the Rational Root Theorem to generate the list of all possible RATIONAL zeros that the function has.
3. Use Descartes Rule of Signs to figure out how many possible POSITIVE/NEGATIVE real zeros we have.
4. Start at the beginning of the list and use synthetic division to find a number that gives a remainder of zero.
5. Repeat 4 until you have at least a quadratic or linear equation to work with then you can use other solving methods we have discussed:
   - Quadratic formula
   - U-Substitution
   - Difference of Squares
   - Solving linear equations

Example

Solve each of the following:

1. $x^3 + 3x^2 - 14x - 20 = 0$

Example

Solve each of the following:

2. $x^6 - 21x^4 + 84x^2 - 64 = 0$

Solving Exponential Equations

Property of emphasis:

Given an equation such as, $b^n = b^m$

Then, $n = m$

Procedure:

1. If needed, rewrite the bases as powers so that they are the same base.
2. Set the exponents equal to each other.
3. Solve for the variable present.

Examples:

1. $8^{x-1} = 2^{x+2}$ (rewrite 8 as 2^3)

So, $(2^3)^{x-1} = 2^{x+2}$

$2^{3(x-1)} = 2^{x+2}$

Meaning solve, $3(x - 1) = x + 2$

2. $(1/3)^{x+2} = 3^{x-1}$ (rewrite 1/3 as 3^{-1})

So, $(3^{-1})^{x+2} = 3^{x-1}$

$3^{-1(x+2)} = 3^{x-1}$

Meaning solve, $-1(x + 2) = x - 1$

Examples

Solve each of the following Exponential Equations:

1. $36^{2x} = 216^{x-1}$

2. $(1/2)^{x} = 2^{x+3}$

Solving Logarithmic Equations

Properties of emphasis:

IF $\log_b m = \log_b n$, then $m = n$
IF $\log_b y = x$, then $y = b^x$
IF $\log_b m + \log_b n$, then $\log_b (mn)$
IF $\log_b m - \log_b n$, then $\log_b (m/n)$

Procedure:

1. Identify the property used from the ones listed above.
2. Apply the property and set up the equation to be solved.
3. Solve the equation using the method required for the situation presented to you.

Examples

Solve each of the following Logarithmic Equations:

1. $\log(10 - 4x) = \log(10 - 3x)$

2. $-10 + \log_3(n + 3) = -10$

More Examples

Solve each of the following Logarithmic Equations:

3. $-2\log_5(7x) = 0$

4. $\log_{12}(v^2 + 35) = \log_{12}(-12v - 1)$

More Examples

Solve each of the following Logarithmic Equations:

5. $\log x + \log 8 = 2$

6. $\ln(-3x - 1) - \ln 7 = 2$ (Note: ln is the same as $\log_e$)