① $x^4 - 25x^2 + 144 = 0$

Rational Root Thm

$q = 144:$ 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144
$p = 1 :$ 1

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 16, \pm 18, \pm 24, \pm 36, \pm 48, \pm 72, \pm 144$

Descartes Rule of Signs

Original has 2 change = 2 or 0 positive roots

Change any odd exponent term sign

All stay the same so 2 change. = 2 or 0 neg. roots

Solve

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -25 & 0 & 144 \\ & \downarrow & 3 & 9 & -48 & -144 \\ \hline & 1 & 3 & -16 & -48 & 0 \end{array}$$

so $\boxed{3 \text{ is a root}}$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -16 & -48 \\ & \downarrow & -3 & 0 & 48 \\ \hline & 1 & 0 & -16 & 0 \end{array}$$

so $\boxed{-3 \text{ is a root}}$

Leaves $x^2 - 16$

Set $x^2 - 16 = 0$

$$\begin{array}{rcl} x^2 - 16 & = & 0 \\ +16 & & +16 \\ \hline \sqrt{x^2} & = & \sqrt{16} \end{array}$$

$$\boxed{x = \pm 4}$$

② $x^3 + 3x^2 - 14x - 20 = 0$

Rational Rt Thm

$q = 20$: 1, 2, 4, 5, 10, 20

$p = 1$: 1

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

Descartes Rule of Signs

Original 1 change

= 1 positive root

Change any odd exponent term sign

$-x^3 + 3x^2 + 14x - 20 = 0$

2 changes

= 2 or 0 negative roots.

Solve:

-5	1	3	-14	-20
	↓	-5	10	20
	1	-2	-4	0

So $\boxed{-5 \text{ is a root}}$

Leaves $x^2 - 2x - 4$

Using Quadratic Formula gives:

$a = 1$

$b = -2$

$c = -4$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$\boxed{x = \frac{2 \pm \sqrt{20}}{2}}$$

OR

$$\boxed{x = 1 \pm \sqrt{5}}$$

③ $x^3 - 2x^2 - 9x + 18 = 0$

Rational Root Thm

$q = 18$: 1, 2, 3, 6, 9, 18
$p = 1$: 1

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

Descartes Rule of Signs

Original 2 changes
= 2 or 0 pos. roots

Change any odd exponent term signs.

$-x^3 - 2x^2 + 9x + 18 = 0$

One change = 1 negative Root.

Solve

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -9 & 18 \\ & \downarrow & 2 & 0 & -18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

So $\boxed{2 \text{ is a root}}$

Leaves $x^2 - 9$

Set $x^2 - 9 = 0$
$\quad +9 \quad +9$

$\sqrt{x^2} = \sqrt{9}$

$\boxed{x = \pm 3}$

④ $x^4 - 34x^2 + 225 = 0$

Rational Root Thm:

$q = 225$: 1, 3, 5, 9, 15, 25, 45, 75, 225

$p = 1$: 1

$\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 25, \pm 45, \pm 75, \pm 225$

Descartes Rule of Signs

Original 2 changes = 2 or 0 positive roots

Change any odd exponent term signs.

Stays the same

2 changes = 2 or 0 negative roots.

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -34 & 0 & 225 \\ & \downarrow & 3 & 9 & -75 & -225 \\ \hline & 1 & 3 & -25 & -75 & 0 \end{array}$$

so $\boxed{3 \text{ is a root}}$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -25 & -75 \\ & \downarrow & -3 & 0 & 75 \\ \hline & 1 & 0 & -25 & 0 \end{array}$$

so $\boxed{-3 \text{ is a root}}$

Leaves $x^2 - 25$

Set $x^2 - 25 = 0$

$ +25 \quad +25$

$\sqrt{x^2} = \sqrt{25}$

$\boxed{x = \pm 5}$

⑤ $x^4 - 20x^2 + 64 = 0$

Rational Root Thm

$q = 64: 1, 2, 4, 8, 16, 32, 64$

$p = 1: 1$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$

Descartes Rule of Signs

Original has 2 changes = 2 or 0 positive roots

Change any odd exponent term signs,

All stay the same so 2 changes = 2 or 0 neg. roots

Solve

Using U-Substitution

Let $u = x^2$

Gives $u^2 - 20u + 64 = 0$

$u^2 - 4u - 16u + 64 = 0$

$(u^2 - 4u) - (16u - 64) = 0$

$u(u-4) - 16(u-16) = 0$

$(u-16)(u-4) = 0$

$1(64) = 64$

1	64
2	32
$\boxed{-4}$	$\boxed{-16}$

Therefore, $(x^2-16)(x^2-4) = 0$

$x^2 - 16 = 0$
$+16 \quad +16$
$\sqrt{x^2} = \sqrt{16}$

$\boxed{x = \pm 4}$

$x^2 - 4 = 0$
$+4 \quad +4$
$\sqrt{x^2} = \sqrt{4}$

$\boxed{x = \pm 2}$

⑥ $x^4 - 45x^2 + 324 = 0$

Rational Root Thm

$q = 324$: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 81, 108, 162, 324

$p = 1$: 1

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 27, \pm 36, \pm 54, \pm 81, \pm 108, \pm 162, \pm 324$

Descartes Rule of Signs

Original 2 changes = 2 or 0 positive roots.

Change any odd exponent term signs

Stays the same so 2 changes = 2 or 0 neg. roots.

Solve:

Using u-Substitution

Let $u = x^2$

Giving $u^2 - 45u + 324 = 0$

$u^2 - 9u - 36u + 324 = 0$

$(u^2 - 9u) - (36u - 324) = 0$

$u(u-9) - 36(u-9) = 0$

$(u-36)(u-9) = 0$

$1(324) = 324$

1	324
2	162
3	108
4	81
6	54
$\boxed{-9}$	$\boxed{-36}$

Therefore, $(x^2-36)(x^2-9) = 0$

$x^2 - 36 = 0$
$+36 \quad +36$

$\sqrt{x^2} = \sqrt{36}$

$\boxed{x = \pm 6}$

$x^2 - 9 = 0$
$+9 \quad +9$

$\sqrt{x^2} = \sqrt{9}$

$\boxed{x = \pm 3}$

⑦ $x^3 + 6x^2 - 9x - 14 = 0$

Rational Root Thm

$q = 14 : 1, 2, 7, 14$

$p = 1 : 1$

$\pm 1, \pm 2, \pm 7, \pm 14$

Descartes Rule of Signs

Original 1 change = 1 pos. root

Change all odd exponent term signs

$-x^3 + 6x^2 + 9x - 14 = 0$

2 changes = 2 or 0 neg. roots

Solve:

$$\begin{array}{r|rrrr} -1 & 1 & 6 & -9 & -14 \\ & \downarrow & -1 & -5 & 14 \\ \hline & 1 & 5 & -14 & 0 \end{array}$$

so $\boxed{-1 \text{ is a root}}$

$$\begin{array}{r|rrr} 2 & 1 & 5 & -14 \\ & \downarrow & 2 & 14 \\ \hline & 1 & 7 & 0 \end{array}$$

so $\boxed{2 \text{ is a root}}$

Leaves $x + 7$

set $x + 7 = 0$

$\quad -7 \quad -7$

$\boxed{x = -7}$

(8) $x^3 + 3x^2 - x - 3 = 0$

Rational Rt Thm

$q = 3: \ 1 \ 3$

$p = 1: \ 1$

$\pm 1, \pm 3$

Descartes Rule of Signs

Original 1 change

= 1 positive root

Change any odd exponent term sign

$-x^3 + 3x^2 + x - 3 = 0$

2 changes

= 2 or 0 negative roots

Solve:

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -1 & -3 \\ & \downarrow & 1 & 4 & 3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

So $\boxed{1 \text{ is a root}}$

$$\begin{array}{r|rrr} -1 & 1 & 4 & 3 \\ & \downarrow & -1 & -3 \\ \hline & 1 & 3 & 0 \end{array}$$

So $\boxed{-1 \text{ is a root}}$

Leaves

$$\begin{array}{rr} x + 3 & = 0 \\ -3 & -3 \\ \hline \end{array}$$

$\boxed{x = -3}$

⑨ $x^3 - 2x^2 - 3x + 6 = 0$

Rational Rt. Thm

$q = 6: \; 1, 2, 3, 6$

$p = 1$

$\pm 1, \pm 2, \pm 3, \pm 6$

Descartes Rule of Signs

Original 2 changes

= 2 or 0 positive roots

Change any odd exponent term signs

$-x^3 - 2x^2 + 3x + 6 = 0$

1 change

= 1 negative root

Solve:

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -3 & 6 \\ & \downarrow & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

So $\boxed{2 \text{ is a root}}$

Leaves $x^2 - 3 = 0$

$\quad +3 \quad +3$

$\sqrt{x^2} = \sqrt{3}$

$$\boxed{x = \pm\sqrt{3}}$$

⑩ $x^6 - 21x^4 + 84x^2 - 64 = 0$

Rational Root Thm

$q = 64: 1, 2, 4, 8, 16, 32, 64$

$p = 1 : 1$

$\pm1, \pm2, \pm4, \pm8, \pm16, \pm32, \pm64$

Descartes Rule of Signs

Original 3 changes

= 3 or 1 positive root

Change all odd exponent term signs

Stays the same 3 changes = 3 or 1 neg. roots

Solve:

$$\begin{array}{r|rrrrrrr} 1 & 1 & 0 & -21 & 0 & 84 & 0 & -64 \\ & \downarrow & 1 & 1 & -20 & -20 & 64 & 64 \\ \hline & 1 & 1 & -20 & -20 & 64 & 64 & 0 \end{array}$$

So $\boxed{1 \text{ is a root}}$

$$\begin{array}{r|rrrrrr} -1 & 1 & 1 & -20 & -20 & 64 & 64 \\ & \downarrow & -1 & 0 & 20 & 0 & -64 \\ \hline & 1 & 0 & -20 & 0 & 64 & 0 \end{array}$$

So $\boxed{-1 \text{ is a root}}$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -20 & 0 & 64 \\ & \downarrow & 2 & 4 & -32 & -64 \\ \hline & 1 & 2 & -16 & -32 & 0 \end{array}$$

So $\boxed{2 \text{ is a root}}$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -16 & -32 \\ & \downarrow & -2 & 0 & 32 \\ \hline & 1 & 0 & -16 & 0 \end{array}$$

So $\boxed{-2 \text{ is a root}}$

Leaving: $x^2 - 16$

set $x^2 - 16 = 0$

$\quad\quad +16 \quad +16$

$\sqrt{x^2} = \sqrt{16}$

$\boxed{x = \pm 4}$

(11) $x^5 + 2x^4 - 45x^3 - 90x^2 + 324x + 648 = 0$

Rational Root Thm

$q = 648$: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72, 81, 108, 162, 216, 324, 648

$p = 1$: 1

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm9, \pm12, \pm18, \pm24, \pm27, \pm36, \pm54, \pm72, \pm81, \pm108, \pm162,$

$\pm216, \pm324, \pm648$

Descartes Rule of Signs

Original 2 changes = 2 or 0 positive roots

Change any odd exponent term signs

$$-x^5 + 2x^4 + 45x^3 - 90x^2 - 324x + 648 = 0$$

3 changes = 3 or 1 neg roots.

Solve

$$\begin{array}{r|rrrrrr} -2 & 1 & 2 & -45 & -90 & 324 & 648 \\ & \downarrow & -2 & 0 & 90 & 0 & -648 \\ \hline & 1 & 0 & -45 & 0 & 324 & \vert\ 0 \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -45 & 0 & 324 \\ & \downarrow & 3 & 9 & -108 & -324 \\ \hline & 1 & 3 & -36 & -108 & \vert\ 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -36 & -108 \\ & \downarrow & -3 & 0 & 108 \\ \hline & 1 & 0 & -36 & \vert\ 0 \end{array}$$

Leaving $x^2 - 36$

Set $x^2 - 36 = 0$

$\quad\quad +36 \quad +36$

$\sqrt{x^2} = \sqrt{36}$

$\boxed{x = \pm 6}$

12) $x^6 - 30x^4 + 129x^2 - 100 = 0$

Rational Root Thm

$q = 100$: 1, 2, 4, 5, 10, 20, 25, 50, 100

$p = 1$: 1

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 25, \pm 50, \pm 100$

Descartes Rule of Signs

Original 3 changes = 3 or 1 positive roots

Change any odd powered exponents

All stay the same so 3 changes = 3 or 1 neg roots

Solve

$$\begin{array}{r|rrrrrrr}
1 & 1 & 0 & -30 & 0 & 129 & 0 & -100 \\
 & \downarrow & 1 & 1 & -29 & -29 & 100 & 100 \\
\hline
 & 1 & 1 & -29 & -29 & 100 & 100 & | \; 0
\end{array}$$

$$\begin{array}{r|rrrrrr}
-1 & 1 & 1 & -29 & -29 & 100 & 100 \\
 & \downarrow & -1 & 0 & 29 & 0 & -100 \\
\hline
 & 1 & 0 & -29 & 0 & 100 & | \; 0
\end{array}$$

$$\begin{array}{r|rrrrr}
2 & 1 & 0 & -29 & 0 & 100 \\
 & \downarrow & 2 & 4 & -50 & -100 \\
\hline
 & 1 & 2 & -25 & -50 & | \; 0
\end{array}$$

$$\begin{array}{r|rrrr}
-2 & 1 & 2 & -25 & -50 \\
 & \downarrow & -2 & 0 & 50 \\
\hline
 & 1 & 0 & -25 & | \; 0
\end{array}$$

Leaves $x^2 - 25$

Set $x^2 - 25 = 0$

$\quad\quad +25 \quad +25$

$\sqrt{x^2} = \sqrt{25}$

$\boxed{x = \pm 5}$

(13) $x^6 - 6x^4 + 8x^3 - 3x^2 = 0$

Factor out as:

$$x^2(x^4 - 6x^2 + 8x - 3) = 0$$

Rational Root Thm

$q = 3 : 1, 3$

$p = 1 : 1$

$\pm 1, \pm 3$

Descartes Rule of Signs

Original 3 changes = 3 or 1 positive roots.

Change any odd exponent terms sign:

$$x^6 - 6x^4 - 8x^3 - 3x^2 = 0$$

One change = 1 neg root

Solve:

Since x^2 can be factored out, 0 and 0 are two of the 6 roots possible

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -6 & 8 & -3 \\ & \downarrow & 1 & 1 & -5 & 3 \\ \hline & 1 & 1 & -5 & 3 & \vert\ 0 \end{array}$$

$$\begin{array}{r|rrr} 1 & 1 & 2 & -3 \\ & \downarrow & 1 & 3 \\ \hline & 1 & 3 & \vert\ 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -5 & 3 \\ & \downarrow & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & \vert\ 0 \end{array}$$

Leaving $x + 3$

set $x + 3 = 0$

$\quad -3 \ \ -3$

$$\boxed{x = -3}$$

(14) $x^8 - 12x^6 + 46x^4 - 60x^2 + 25 = 0$

Rational Root Thm

$q = 25 : 1, 5, 25$

$p = 1 : 1$

$\pm 1, \pm 5, \pm 25$

Descartes Rule of Signs

Original 4 changes = 4, 2, or 0 pos. roots

Change all odd exponents

All stay the same

4, 2, 0 neg roots.

Solve

$$\begin{array}{r|rrrrrrrrr} 1 & 1 & 0 & -12 & 0 & 46 & 0 & -60 & 0 & 25 \\ & \downarrow & 1 & 1 & -11 & -11 & & 35 & -25 & -25 \\ \hline & 1 & 1 & -11 & -11 & 35 & 35 & -25 & -25 & | \; 0 \end{array}$$

$$\begin{array}{r|rrrrrrrr} -1 & 1 & 1 & -11 & -11 & 35 & 35 & -25 & -25 \\ & \downarrow & -1 & 0 & 11 & 0 & -35 & 0 & 25 \\ \hline & 1 & 0 & -11 & 0 & 35 & 0 & -25 & | \; 0 \end{array}$$

$$\begin{array}{r|rrrrrrr} 1 & 1 & 0 & -11 & 0 & 35 & 0 & -25 \\ & \downarrow & 1 & 1 & -10 & -10 & 25 & 25 \\ \hline & 1 & 1 & -10 & -10 & 25 & 25 & | \; 0 \end{array}$$

$$\begin{array}{r|rrrrrr} -1 & 1 & 1 & -10 & -10 & 25 & 25 \\ & \downarrow & -1 & 0 & 10 & 0 & -25 \\ \hline & 1 & 0 & -10 & 0 & 25 & | \; 0 \end{array}$$

Leaves

$x^4 - 10x^2 + 25$

Set

$u = x^2$

$u^2 - 10u + 25 = 0$ $\quad \begin{array}{cc} \multicolumn{2}{c}{25} \\ 1 & 25 \\ 5 & 5 \end{array}$

$u^2 - 5u - 5u + 25 = 0$

$(u^2 - 5u) - (5u - 25) = 0$

$u(u-5) - 5(u-5) = 0$

$(u-5)(u-5) = 0$

$(x^2-5)(x^2-5) = 0$

$$\begin{array}{rr} x^2 - 5 = 0 \\ +5 \quad +5 \\ \hline \sqrt{x^2} = \sqrt{5} \end{array} \qquad \begin{array}{rr} x^2 - 5 = 0 \\ +5 \quad +5 \\ \hline \sqrt{x^2} = \sqrt{5} \end{array}$$

$\boxed{x = \pm\sqrt{5}}$ $\qquad$ $\boxed{x = \pm\sqrt{5}}$

15. $x^3 + x^2 - 101x + 99 = 0$

Rational Root Thm

$q = 99$: 1, 3, 9, 11, 33, 99

$p = 1$

$\pm 1, \pm 3, \pm 9, \pm 11, \pm 33, \pm 99$

Descartes Rule of Signs

Original 2 changes = 2 or 0 pos. root.

Change all odd exponent term signs

$-x^3 + x^2 + 101x + 99 = 0$

1 change = 1 neg root.

Solve:

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -101 & 99 \\ & \downarrow & 1 & 2 & -99 \\ \hline & 1 & 2 & -99 & \multicolumn{1}{|r}{0} \end{array}$$

So $\boxed{1 \text{ is a root}}$

$$\begin{array}{r|rrr} 9 & 1 & 2 & -99 \\ & \downarrow & 9 & 99 \\ \hline & 1 & 11 & \multicolumn{1}{|r}{0} \end{array}$$

So $\boxed{9 \text{ is a root}}$

Leaves $x + 11$

Set $x + 11 = 0$

$$\begin{array}{rcr} x + 11 & = & 0 \\ -11 & & -11 \\ \hline \end{array}$$

$\boxed{x = -11}$

(16) $x^3 + 2x^2 - 9x - 4 = 0$

Rational Rt. Thm

$q = 4: \ 1, 2, 4$

$p = 1: \ 1$

$\pm 1, \pm 2, \pm 4$

Descartes Rule of Signs

Original 1 change

= 1 positive root

Change any odd exponent term signs

$-x^3 + 2x^2 + 9x - 4 = 0$

2 changes

= 2 or 0 negative roots

Solve:

$$\begin{array}{r|rrrr} -4 & 1 & 2 & -9 & -4 \\ & \downarrow & -4 & 8 & 4 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

So $\boxed{-4 \text{ is a root}}$

Leaves $1x^2 - 2x - 1 = 0$

Using Quadratic Formula

$a = 1$

$b = -2$

$c = -1$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$\boxed{x = \frac{2 \pm \sqrt{8}}{2}}$$

OR

$$\boxed{x = 1 \pm \sqrt{2}}$$

(17) $x^4 + 2x^3 - 7x^2 - 6x + 12 = 0$

Rational Rt. Thm

$q = 12 : 1, 2, 3, 4, 6, 12$

$p = 1 : 1$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Descartes Rule of Signs

Original 2 changes

= 2 or 0 positive roots

Change any odd exponent term signs

$x^4 - 2x^3 - 7x^2 + 6x + 12 = 0$

2 changes

= 2 or 0 negative roots

Solve

$(x^2 - 3)(x^2 + 2x - 4)$

$x^2 - 3 = 0$

$+3 \quad +3$

$\sqrt{x^2} = \sqrt{3}$

$\boxed{x = \pm\sqrt{3}}$

$x^2 + 2x - 4 = 0$

$a = 1$

$b = 2$

$c = -4$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$

$x = \boxed{\dfrac{-2 \pm \sqrt{20}}{2}}$

OR

$\boxed{x = -1 \pm \sqrt{5}}$

(18) $x^4 - 2x^3 - 10x^2 + 4x + 16 = 0$

Rational Root Thm

$q = 16 : 1, 2, 4, 8, 16$

$p = 1 : 1$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Descartes Rule of Signs

Original 2 changes = 2 or 0 pos.

Change all odd exponent term sign

$x^4 + 2x^3 - 10x^2 - 4x + 16 = 0$

2 changes = 2 or 0 neg roots.

Solve:

-2	1	-2	-10	4	16
	↓	-2	8	4	-16
	1	-4	-2	8	0

So $\boxed{-2 \text{ is a root}}$

4	1	-4	-2	8
	↓	4	0	-8
	1	0	-2	0

So $\boxed{4 \text{ is a root}}$

Leaves $x^2 - 2$

set $x^2 - 2 = 0$

$\quad +2 \quad +2$

$\sqrt{x^2} = \sqrt{2}$

$\boxed{x = \pm\sqrt{2}}$

(19) $x^4 - 8x^3 - 37x^2 + 380x - 672 = 0$

Rational Root Thm

$q = 672$: 1, 2, 3, 4, 6, 7, 8, 12, 14, 16, 21, 24, 28, 32, 42, 48, 56, 84, 96, 112, 168, 224, 336, 672

$p = 1$: 1

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 7, \pm 8, \pm 12, \pm 14, \pm 16, \pm 21, \pm 24, \pm 28, \pm 32, \pm 42, \pm 48, \pm 56, \pm 84, \pm 96, \pm 112, \pm 168, \pm 224, \pm 336, \pm 672$

Descartes Rule of Signs

Original 3 changes = 3 or 1 positive roots

Change any odd exponent term signs

$x^4 + 8x^3 - 37x^2 - 380x - 672 = 0$

1 change = 1 negative Root.

Solve

$$\begin{array}{r|rrrr|r} 3 & 1 & -8 & -37 & 380 & -672 \\ & \downarrow & 3 & -15 & -156 & 672 \\ \hline & 1 & -5 & -52 & 224 & 0 \end{array}$$

$$\begin{array}{r|rrr|r} 4 & 1 & -5 & -52 & 224 \\ & \downarrow & 4 & -4 & -224 \\ \hline & 1 & -1 & -56 & 0 \end{array}$$

$$\begin{array}{r|rr|r} -7 & 1 & -1 & -56 \\ & \downarrow & -7 & 56 \\ \hline & 1 & -8 & 0 \end{array}$$

Leaving $x - 8$

Set $x - 8 = 0$

$\quad\quad +8 \quad +8$

$\boxed{x = 8}$

20. $x^4 + 2x^3 + 2x - 1 = 0$

Rational Rt. Thm

$q = 1 : 1$

$p = 1 : 1$

± 1

Descartes Rule of Signs

Original 1 change

= 1 positive root

Change any odd exponent term sign

$x^4 - 2x^3 - 2x - 1 = 0$

1 change

= 1 negative Root

Solve:

$$\left[(x+1)^2 - 2\right]^2 = 0$$

$$(x+1)^2 - 2 = 0$$
$$+2 \qquad +2$$
$$\sqrt{(x+1)^2} = \sqrt{2}$$
$$x+1 = \pm\sqrt{2}$$
$$-1 \qquad -1$$
$$\boxed{x = -1 \pm \sqrt{2}}$$

$$(x+1)^2 - 2 = 0$$
$$+2 \qquad +2$$
$$\sqrt{(x+1)^2} = \sqrt{2}$$
$$x+1 = \pm\sqrt{2}$$
$$-1 \qquad -1$$
$$\boxed{x = -1 \pm \sqrt{2}}$$