

Properties:

(A) $\log_b y = x \Leftrightarrow b^x = y$

(C) $\log \frac{m}{n} = \log m - \log n$

(B) $\log mn = \log m + \log n$

(D) $\log x = \log y \Leftrightarrow x = y$

Solving Equations

Emphasis solving Logarithmic Equations

Solve each of the following Logarithmic Equations:

1. $\log 5x = \log(2x + 9)$

$$\begin{array}{r} 5x = 2x + 9 \\ -2x \quad -2x \\ \hline 3x = 9 \\ \frac{3x}{3} = \frac{9}{3} \end{array} \quad \boxed{x = 3}$$

2. $\log(10 - 4x) = \log(10 - 3x)$

$$\begin{array}{r} 10 - 4x = 10 - 3x \\ +4x \quad \quad +4x \\ \hline 10 = 10 + x \\ -10 \quad -10 \\ \hline 0 = x \end{array} \quad \boxed{0 = x}$$

3. $\log(4p - 2) = \log(-5p + 5)$

$$\begin{array}{r} 4p - 2 = -5p + 5 \\ +5p \quad \quad +5p \\ \hline 9p - 2 = 5 \\ +2 \quad +2 \\ \hline 9p = 7 \\ \frac{9p}{9} = \frac{7}{9} \end{array} \quad \boxed{p = \frac{7}{9}}$$

4. $\log(4k - 5) = \log(2k - 1)$

$$\begin{array}{r} 4k - 5 = 2k - 1 \\ -2k \quad \quad -2k \\ \hline 2k - 5 = -1 \\ +5 \quad +5 \\ \hline 2k = 4 \\ \frac{2k}{2} = \frac{4}{2} \end{array} \quad \boxed{k = 2}$$

5. $\log(-2a + 9) = \log(7 - 4a)$

$$\begin{array}{r} -2a + 9 = 7 - 4a \\ +4a \quad \quad +4a \\ \hline 2a + 9 = 7 \\ -9 \quad -9 \\ \hline 2a = -2 \\ \frac{2a}{2} = \frac{-2}{2} \end{array} \quad \boxed{a = -1}$$

6. $2 \log_7(-2r) = 0$

$$\begin{array}{r} \frac{2}{2} \log_7(-2r) = \frac{0}{2} \\ \log_7(-2r) = 0 \\ -2r = 7^0 \\ \frac{-2r}{-2} = \frac{1}{-2} \end{array} \quad \boxed{r = -\frac{1}{2}}$$

7. $-10 + \log_3(n + 3) = -10$

$$\begin{array}{r} -10 + \log_3(n + 3) = -10 \\ +10 \quad \quad +10 \\ \hline \log_3(n + 3) = 0 \\ n + 3 = 3^0 \\ n + 3 = 1 \\ -3 \quad -3 \\ \hline n = -2 \end{array} \quad \boxed{n = -2}$$

8. $-2 \log_5(7x) = 0$

$$\begin{array}{r} \frac{-2}{-2} \log_5(7x) = \frac{0}{-2} \\ \log_5(7x) = 0 \\ 7x = 5^0 \\ \frac{7x}{7} = \frac{1}{7} \end{array} \quad \boxed{x = \frac{1}{7}}$$

9. $\log(-m) + 2 = 4$

$$\begin{array}{r} \log(-m) + 2 = 4 \\ -2 \quad -2 \\ \hline \log(-m) = 2 \\ -m = 10^2 \\ \frac{-m}{-1} = \frac{100}{-1} \end{array} \quad \boxed{m = -100}$$

10. $-6 \log_3(x - 3) = -24$

$$\begin{array}{r} \frac{-6}{-6} \log_3(x - 3) = \frac{-24}{-6} \\ \log_3(x - 3) = 4 \\ x - 3 = 3^4 \\ x - 3 = 81 \\ +3 \quad +3 \\ \hline x = 84 \end{array} \quad \boxed{x = 84}$$

$$11. \log_{12}(v^2 + 35) = \log_{12}(-12v - 1)$$

$$\begin{aligned} v^2 + 35 &= -12v - 1 \\ +12v + 1 &+12v + 1 \\ \hline v^2 + 12v + 36 &= 0 \\ (v^2 + 6v) + (6v + 36) &= 0 \\ (v+6) + 6(v+6) &= 0 \\ (v+6)(v+6) &= 0 \end{aligned}$$

$$\begin{aligned} v+6 &= 0 \\ -6 &-6 \\ \hline v &= -6 \end{aligned}$$

$$\begin{aligned} v+6 &= 0 \\ -6 &-6 \\ \hline v &= -6 \end{aligned}$$

$$12. \log_9(-11x + 2) = \log_9(x^2 + 30)$$

$$\begin{aligned} -11x + 2 &= x^2 + 30 \\ 0 &= x^2 + 11x + 28 \\ 0 &= (x+7)(x+4) \end{aligned}$$

$$\begin{aligned} x+7 &= 0 \\ -7 &-7 \\ \hline x &= -7 \end{aligned}$$

$$\begin{aligned} x+4 &= 0 \\ -4 &-4 \\ \hline x &= -4 \end{aligned}$$

$$13. \log(16 + 2b) = \log(b^2 - 4b)$$

$$\begin{aligned} 16 + 2b &= b^2 - 4b \\ 0 &= b^2 - 6b - 16 \\ 0 &= (b-8)(b+2) \end{aligned}$$

$$\begin{aligned} b-8 &= 0 \\ +8 &+8 \\ \hline b &= 8 \end{aligned}$$

$$\begin{aligned} b+2 &= 0 \\ -2 &-2 \\ \hline b &= -2 \end{aligned}$$

$$14. \ln(n^2 + 12) = \ln(-9n - 2)$$

$$\begin{aligned} n^2 + 12 &= -9n - 2 \\ n^2 + 9n + 14 &= 0 \\ (n+7)(n+2) &= 0 \end{aligned}$$

$$\begin{aligned} n+7 &= 0 \\ -7 &-7 \\ \hline n &= -7 \end{aligned}$$

$$\begin{aligned} n+2 &= 0 \\ -2 &-2 \\ \hline n &= -2 \end{aligned}$$

$$15. \log x + \log 8 = 2$$

$$\begin{aligned} \log 8x &= 2 \\ 8x &= 10^2 \\ \frac{8x}{8} &= \frac{100}{8} \end{aligned}$$

$$x = \frac{25}{2}$$

$$16. \log x - \log 2 = 1$$

$$\begin{aligned} \log \frac{x}{2} &= 1 \\ 2 \cdot \left[\frac{x}{2} = 10^1 \right] \cdot 2 \end{aligned}$$

$$x = 20$$

$$17. \log 2 + \log x = 1$$

$$\begin{aligned} \log 2x &= 1 \\ 2x &= 10^1 \\ \frac{2x}{2} &= \frac{10}{2} \end{aligned}$$

$$x = 5$$

$$18. \log x + \log 7 = \log 37$$

$$\begin{aligned} \log 7x &= \log 37 \\ \frac{7x}{7} &= \frac{37}{7} \end{aligned}$$

$$x = \frac{37}{7}$$

$$19. \ln 2 - \ln(3x + 2) = 1$$

$$\begin{aligned} \ln \frac{2}{3x+2} &= 1 \\ \log_e \frac{2}{3x+2} &= 1 \\ \frac{2}{3x+2} &= e^1 \end{aligned}$$

$$\begin{aligned} 2 &= 3ex + 2e \\ -2e &-2e \\ \hline 2-2e &= 3ex \\ \frac{2-2e}{3e} &= \frac{3ex}{3e} \end{aligned}$$

$$\frac{2}{3e} - \frac{2}{3} = x$$

$$20. \ln(-3x - 1) - \ln 7 = 2$$

$$\begin{aligned} \ln \frac{-3x-1}{7} &= 2 \\ \log_e \frac{-3x-1}{7} &= 2 \\ \frac{-3x-1}{7} &= e^2 \\ -3x-1 &= 7e^2 \\ +1 &+1 \end{aligned}$$

$$\begin{aligned} -3x &= 7e^2 + 1 \\ -3 &-3 \\ \hline x &= \frac{7e^2 + 1}{-3} \end{aligned}$$