

Bellwork

Multiply the following polynomials:

1. $(x - 5)(x + 5)$

$$x(x+5) - 5(x+5)$$

$$x^2 + 5x - 5x - 25$$

$$x^2 - 25$$

2. $(x - 1)(x - 1)$

$$x(x-1) - 1(x-1)$$

$$x^2 - 1x - 1x + 1$$

$$x^2 - 2x + 1$$

3. $(x + 9)(x + 9)$

$$x(x+9) + 9(x+9)$$

$$x^2 + 9x + 9x + 81$$

$$x^2 + 18x + 81$$

4. $(x + 6)(x - 4)$

$$x(x-4) + 6(x-4)$$

$$x^2 - 4x + 6x - 24$$

$$x^2 + 2x - 24$$

Recall the Distributive Property

In the notes about the Distributive Property we showed the property when multiplying two binomials, two trinomials, or a mix of the two of them.

Property of Focus:

$$\begin{aligned}(x + a)(x + b) &= x(x + b) + a(x + b) \\ &= x^2 + bx + ax + ab \\ &= x^2 + ax + bx + ab \\ &= x^2 + (a+b)x + ab\end{aligned}$$

Investigation of the Trinomial Result

Property of Focus:

$$\begin{aligned}(x + a)(x + b) &= x(x + b) + a(x + b) \\ &= x^2 + bx + ax + ab \\ &= x^2 + ax + bx + ab \\ &= x^2 + (a+b)x + ab\end{aligned}$$

What do you notice about the middle term of the end result here?

It comes from adding the two terms that are constants.

What do you notice about the last term, the constant, of the end result here?

It comes from multiplying the two terms that are constants.

Factoring Trinomial Expressions

Standard Form:

$$ax^2 + bx + c$$

Focus:

We will focus on the case where $a = 1$

So we are really looking at...

$$x^2 + bx + c$$

Procedure

Step 1: Create the "U" to find **ALL** of the factors of the constant C.

Step 2: Identify the set of factors that combine by the **SECOND** sign to get the middle number.

We will let these factors be p and q.

Step 3: Write the factored version of your trinomial.

$$(x + p)(x + q)$$

Step 4: **CHECK YOUR ANSWER!**

Worked Out Examples

Factor each of the following expressions:

1. $x^2 - 8x - 20$

Find the factors 20:

1 and 20

2 and 10

4 and 5

The second sign is - so the factors must subtract to get - 8.

$$\text{So } 2 - 10 = - 8$$

Answer: $(x + 2)(x - 10)$

2. $x^2 - 15x + 50$

Find the factors of 50:

1 and 50

2 and 25

5 and 10

The second sign is + so the factors must add to get -15.

$$\text{So } -5 + (-10) = -15$$

Answer: $(x - 10)(x - 5)$

Common Errors

Find the error in this example:

Ex) Factor $x^2 - 10x + 24$

Factors of 24

1 and 24

2 and 12

3 and 8

4 and 6

Well 12 minus 2 will give us 10...

So the Answer has to be **$(x + 12)(x - 2)$**

Examples

Factor each of the following expressions:

1. $x^2 - 4x - 165$

Find the factors of 165:

1 and 165

3 and 55

5 and 33

11 and 15

The second sign is - so the factors must subtract to get - 4.

$$\text{So } 11 - 15 = -4$$

Answer: $(x + 11)(x - 15)$

2. $x^2 - 6x - 72$

Find the factors of 72:

1 and 72

2 and 36

3 and 24

4 and 18

6 and 12

8 and 9

The second sign is - so the factors must subtract to get -6.

$$\text{So } 6 - 12 = -6$$

Answer: $(x - 12)(x + 6)$

Example

Factor each of the following expressions:

3. $x^2 + 28x + 196$

Find the factors of 196:

- 1 and 196
- 2 and 98
- 4 and 49
- 14 and 14

The second sign is + so the factors must add to get 28.

$$\text{So } 14 + 14 = 28$$

Answer: $(x + 14)(x + 14)$

OR
 $(x + 14)^2$

4. $x^2 - 169$

Find the factors of 169:

- 1 and 169
- 13 and 13

Rewrite as:

$$x^2 + 0x - 169$$

The second sign is - so the factors must subtract to get 0.

$$\text{So } 13 - 13 = 0$$

Answer: $(x + 13)(x - 13)$