

Factoring Polynomials

There are several factoring methods that we will cover:

1. Factoring out a Greatest Common Factor (GCF)
2. Factoring a Difference of Squares
3. Factoring Quadratics
 - A. With leading coefficient equal to 1 ($a = 1$)
 - B. With leading coefficient not equal to 1 ($a \neq 1$)
4. Factoring using a method referred to as U-Substitution
5. Factoring a Sum of Cubes
6. Factoring a Difference of Cubes
7. Factoring by Grouping

Factoring out a GCF

Procedure:

1. Factor each term out completely.
(This can be physically written or just mentally done)
2. Remove anything that ALL terms have in common.
3. Multiply all the items identified in step 2 together.
4. Place the product from Step 3 in front of a set of parentheses that contains the remaining items from each of the terms.

Examples:

1. $7x^4 - 14x^2 + 28x$

$7x^4 = 7 \times x \times x \times x$

$-14x^2 = -1 (2) (7) \times x \times x$

$28x = 2 (2) (7) \times x$

Answer:

$7x(x^3 - 2x + 4)$

2. $15x^3y + 10x^2y^2 - 25xy^3$

$15x^3y = 3 (5) \times x \times x \times x \times y$

$10x^2y^2 = 2 (5) \times x \times x \times y \times y$

$-25xy^3 = -1 (5) (5) \times y \times y \times y$

Answer:

$5xy(3x^2 + 2xy - 5y^2)$

Examples

1. $6x^2 - 24x$

2. $9x^7y^5 - 3x^2y^6$

Factoring out a Difference of Squares

What is a Difference of Squares?

This is an expression of the type $a^2 - b^2$

Where each of the terms are perfect squares!

General Property:

$$a^2 - b^2 = (a - b)(a + b)$$

Procedure:

1. Find the square root of the first term.
2. Find the square root of the second term.
3. Write (Step 1 + Step 2)(Step 1 - Step 2)

Examples:

1. $x^2 - 400$

$$\sqrt{x^2} = x$$

$$\sqrt{400} = 20$$

Answer:

$$(x + 20)(x - 20)$$

2. $49x^2 - 25$

$$\sqrt{49x^2} = 7x$$

$$\sqrt{25} = 5$$

Answer:

$$(7x + 5)(7x - 5)$$

Examples

1. $x^2 - 441$

2. $4x^2 - 25$

3. $121x^2 - 900$

Factoring by using U-Substitution

What is U-Substitution?

This is a method of taking a trinomial that is not a quadratic, and making it appear to be a quadratic.

General Property:

$$ax^{2n} + bx^{1n} + c$$

$$\text{Letting } U = x^n$$

$$\text{Giving us } au^2 + bu + c$$

Procedure:

1. Find out what you are going to allow U to equal.
(Typically it will be the variable on the middle term)
2. Rewrite the expression using u now.
3. Factor out like a typical quadratic expression.
4. Replace u and check the factors that you have and factor them out more as needed.

Examples

1. $x^4 - 9x^2 + 20$

2. $2x^4 + 9x^2 - 5$

Another Example

3. $b - 20\sqrt{b} + 64$

Factoring out a Sum/Difference of Cubes

What is a Sum/Difference of Cubes?

This is an expression of the type $a^3 - b^3$

Where each of the terms are perfect cubes!

General Properties:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Procedure:

1. Find the cube root of the first term.
2. Find the cube root of the second term.
3. Remember your S-O-AP and plug into the property.

S = Same sign

O = Opposite sign

AP = Always positive

$$(a \pm b)(a^2 \mp ab + b^2)$$

Sum of Cubes Examples

1. $x^3 + 8$

2. $27x^3 + 729$

Difference of Cubes Examples

1. $x^3 - 64$

2. $1331x^3 - 729$

Factoring by Grouping

What is Factoring by Grouping?

When given a polynomial of 4 terms in standard form, sometimes you can factor the expression by grouping the first two terms together and finding the GCF, and repeating the process for the second set of terms.

If what is in () after finding the GCF does not match then one of two things could be wrong:

1. You can't factor using factor by grouping.
2. You didn't pull enough out for the GCF of one set.

Procedure:

1. Place parentheses around the first two terms.
2. Leave the sign in the middle and place parentheses around the last two terms. NOTE: If sign in the middle is negative then change the sign between the last two terms.
3. Factor each set of parentheses by finding the GCF of each.
4. Rewrite as (GCF first set + GCF second set)(Like set remaining)
Example: $2x^2(x - 5) - 7(x - 5)$ becomes $(2x^2 - 7)(x - 5)$
5. Check each factor for more factoring possibilities.

Examples

1. $25v^3 + 5v^2 + 30v + 6$

2. $5n^3 - 10n^2 + 3n - 6$