Factoring Polynomials

Emphasis on Factoring Trinomials using U-Substitution

Completely factor each of the following using U-Substitution:

1. $x^4 - 9x^2 + 20$

$(1)(20) = 20$: $1\ 20$, $2\ 10$, $-4\ -5$

$u = x^2$

$u^2 - 9u + 20$

$u^2 - 4u - 5u + 20$

$(u^2 - 4u) - (5u - 20)$

$u(u-4) - 5(u-4)$

$(u-5)(u-4)$

$(x^2-5)(x^2-4) = \boxed{(x^2-5)(x+2)(x-2)}$

2. $t^4 - 7t^2 + 12$

$1(12) = 12$: $1\ 12$, $2\ 6$, $-3\ -4$

$u = t^2$

$u^2 - 7u + 12$

$u^2 - 3u - 4u + 12$

$(u^2 - 3u) - (4u - 12)$

$u(u-3) - 4(u-3)$

$(u-4)(u-3)$

$(t^2-4)(t^2-3) = \boxed{(t-2)(t+2)(t^2-3)}$

3. $x^4 + x^2 - 12$

$1(12) = 12$: $1\ 12$, $2\ 6$, $-3\ +4$

$u = x^2$

$u^2 + u - 12$

$u^2 - 3u + 4u - 12$

$(u^2 - 3u) + (4u - 12)$

$u(u-3) + 4(u-3)$

$(u+4)(u-3)$

$\boxed{(x^2+4)(x^2-3)}$

4. $x^4 - 7x^2 - 18$

$1(18) = 18$: $1\ 18$, $+2\ -9$

$u = x^2$

$u^2 - 7u - 18$

$u^2 + 2u - 9u - 18$

$(u^2 + 2u) - (9u + 18)$

$u(u+2) - 9(u+2)$

$(u-9)(u+2)$

$(x^2-9)(x^2+2) = \boxed{(x+3)(x-3)(x^2+2)}$

5. $2w^4 - 9w^2 + 4$

$2(4) = 8$: $-1\ -8$, $2\ 4$

$u = w^2$

$2u^2 - 9u + 4$

$2u^2 - 1u - 8u + 4$

$(2u^2 - 1u) - (8u - 4)$

$u(2u-1) - 4(2u-1)$

$(u-4)(2u-1)$

$(w^2-4)(2w^2-1) = \boxed{(w-2)(w+2)(2w^2-1)}$

6. $3x^4 - 5x^2 + 2$

$3(2) = 6$: $1\ 6$, $-2\ -3$

$u = x^2$

$3u^2 - 5u + 2$

$3u^2 - 2u - 3u + 2$

$(3u^2 - 2u) - (3u - 2)$

$u(3u-2) - 1(3u-2)$

$(u-1)(3u-2)$

$(x^2-1)(3x^2-2) = \boxed{(x+1)(x-1)(3x^2-2)}$

7. $x^4 - 4x^2 + 4$

$1(4) = 4$: $1\ 4$, $-2\ -2$

$u = x^2$

$u^2 - 4u + 4$

$u^2 - 2u - 2u + 4$

$(u^2 - 2u) - (2u - 4)$

$u(u-2) - 2(u-2)$

$(u-2)(u-2)$

$\boxed{(x^2-2)(x^2-2)}$ OR $\boxed{(x^2-2)^2}$

8. $y^4 - 6y^2 + 9$

$1(9) = 9$: $1\ 9$, $-3\ -3$

$u = y^2$

$u^2 - 6u + 9$

$u^2 - 3u - 3u + 9$

$(u^2 - 3u) - (3u - 9)$

$u(u-3) - 3(u-3)$

$(u-3)(u-3) = \boxed{(y^2-3)(y^2-3)}$

OR $\boxed{(y^2-3)^2}$

9. $3x^4 + 16x^2 - 12$

$3(12) = 36$: $1\ 36$, $-2\ +18$, $3\ 12$, $4\ 9$, $6\ 6$

$u = x^2$

$3u^2 + 16u - 12$

$3u^2 - 2u + 18u - 12$

$(3u^2 - 2u) + (18u - 12)$

$u(3u-2) + 6(3u-2)$

$(u+6)(3u-2)$

$\boxed{(x^2+6)(3x^2-2)}$

10. $2x^4 + 9x^2 - 5$

$2(5) = 10$: $-1\ +10$, $2\ 5$

$u = x^2$

$2u^2 + 9u - 5$

$2u^2 - 1u + 10u - 5$

$(2u^2 - 1u) + (10u - 5)$

$u(2u-1) + 5(2u-1)$

$(u+5)(2u-1) = \boxed{(x^2+5)(2x^2-1)}$

11. $2z^4 + 4z^2 - 70$

$1(35) = 35$: $1, 35$; $-5 + 7$

$u = z^2$

$\frac{2z^4}{2} + \frac{4z^2}{2} - \frac{70}{2}$

$u^2 + 2u - 35$

$u^2 - 5u + 7u - 35$

$(u^2 - 5u) + (7u - 35)$

$u(u-5) + 7(u-5)$

$(u-5)(u+7)$

$2(z^2-5)(z^2+7)$

12. $3y^4 + 10y^2 - 8$

$3(8) = 24$: $1, 24$; $-2 + 12$; $3, 8$; $4, 6$

$u = y^2$

$3u^2 + 10u - 8$

$3u^2 - 2u + 12u - 8$

$(3u^2 - 2u) + (12u - 8)$

$u(3u-2) + 4(3u-2)$

$(u+4)(3u-2)$

$(y^2+4)(3y^2-2)$

13. $4t^4 - 20t^2$

$\frac{4t^4}{4t^2} - \frac{20t^2}{4t^2}$

$t^2 - 5$

$4t^2(t^2-5)$

14. $r^4 - 81$

$(r^2-9)(r^2+9)$

$(r-3)(r+3)(r^2+9)$

$u = p^2$

15. $x^4 - x^2 - 12$

$1(12) = 12$: $1, 12$; $2, 6$; $+3 - 4$

$u = x^2$

$u^2 - u - 12$

$u^2 + 3u - 4u - 12$

$(u^2 + 3u) - (4u + 12)$

$u(u+3) - 4(u+3)$

$(u-4)(u+3)$

$(x^2-4)(x^2+3) = (x+2)(x-2)(x^2+3)$

16. $w^4 + w^2 - 12$

$1(12) = 12$: $1, 12$; $2, 6$; $-3 + 4$

$u = w^2$

$u^2 + u - 12$

$u^2 - 3u + 4u - 12$

$(u^2 - 3u) + (4u - 12)$

$u(u-3) + 4(u-3)$

$(u+4)(u-3)$

$(w^2+4)(w^2-3)$

17. $2y^4 + y^2 - 15$

$2(15) = 30$: $1, 30$; $2, 15$; $3, 10$; $-5 + 6$

$u = y^2$

$2u^2 + u - 15$

$2u^2 - 5u + 6u - 15$

$(2u^2 - 5u) + (6u - 15)$

$u(2u-5) + 3(2u-5)$

$(u+3)(2u-5)$

$(y^2+3)(2y^2-5)$

18. $4x^4 - 5x^2 - 9$

$4(9) = 36$: $1, 36$; $2, 18$; $3, 12$; $+4 - 9$; $6, 6$

$u = x^2$

$4u^2 - 5u - 9$

$4u^2 + 4u - 9u - 9$

$(4u^2 + 4u) - (9u + 9)$

$4u(u+1) - 9(u+1)$

$(4u-9)(u+1)$

$(4x^2-9)(x^2+1) = (2x+3)(2x-3)(x^2+1)$

19. $b - 20\sqrt{b} + 64$

$1(64) = 64$: $1, 64$; $2, 32$; $-4 - 16$; $8, 8$

$u = \sqrt{b}$

$u^2 - 20u + 64$

$u^2 - 4u - 16u + 64$

$(u^2 - 4u) - (16u - 64)$

$u(u-4) - 16(u-4)$

$(u-16)(u-4)$

$(\sqrt{b}-16)(\sqrt{b}-4)$

20. $z - 6\sqrt{z} + 8$

$1(8) = 8$: $1, 8$; $-2 - 4$

$u = \sqrt{z}$

$u^2 - 6u + 8$

$u^2 - 2u - 4u + 8$

$(u^2 - 2u) - (4u - 8)$

$u(u-2) - 4(u-2)$

$(u-4)(u-2)$

$(\sqrt{z}-4)(\sqrt{z}-2)$