

## Factoring Polynomials

Emphasis on Factoring Trinomials using U-Substitution

Completely factor each of the following using U-Substitution:

<p>1. <math>x^4 - 9x^2 + 20</math>  <math>u^2 - 9u + 20</math>  <math>u^2 - 4u - 5u + 20</math>  <math>(u^2 - 4u) - (5u - 20)</math>  <math>u(u - 4) - 5(u - 4)</math>  <math>(u - 5)(u - 4)</math>  <math>(x^2 - 5)(x^2 - 4) = (x^2 - 5)(x + 2)(x - 2)</math></p> <p style="text-align: right;"><math>(1)(20) = 20</math>  <math>\begin{array}{r} 1 \ 20 \\ 2 \ 10 \\ \hline -4 \ -5 \end{array}</math></p>	<p>2. <math>t^4 - 7t^2 + 12</math>  <math>u^2 - 7u + 12</math>  <math>u^2 - 3u - 4u + 12</math>  <math>(u^2 - 3u) - (4u - 12)</math>  <math>u(u - 3) - 4(u - 3)</math>  <math>(u - 4)(u - 3)</math>  <math>(t^2 - 4)(t^2 - 3) = (t - 2)(t + 2)(t^2 - 3)</math></p> <p style="text-align: right;"><math>1(12) = 12</math>  <math>\begin{array}{r} 1 \ 12 \\ 2 \ 6 \\ \hline -3 \ -4 \end{array}</math></p>
<p>3. <math>x^4 + x^2 - 12</math>  <math>u^2 + u - 12</math>  <math>u^2 - 3u + 4u - 12</math>  <math>(u^2 - 3u) + (4u - 12)</math>  <math>u(u - 3) + 4(u - 3)</math>  <math>(u + 4)(u - 3)</math>  <math>(x^2 + 4)(x^2 - 3)</math></p> <p style="text-align: right;"><math>1(12) = 12</math>  <math>\begin{array}{r} 1 \ 12 \\ 2 \ 6 \\ \hline -3 \ +4 \end{array}</math></p>	<p>4. <math>x^4 - 7x^2 - 18</math>  <math>u^2 - 7u - 18</math>  <math>u^2 + 2u - 9u - 18</math>  <math>(u^2 + 2u) - (9u + 18)</math>  <math>u(u + 2) - 9(u + 2)</math>  <math>(u - 9)(u + 2)</math>  <math>(x^2 - 9)(x^2 + 2) = (x + 3)(x - 3)(x^2 + 2)</math></p> <p style="text-align: right;"><math>1(18) = 18</math>  <math>\begin{array}{r} 1 \ 18 \\ \hline +2 \ -9 \end{array}</math></p>
<p>5. <math>2w^4 - 9w^2 + 4</math>  <math>2u^2 - 9u + 4</math>  <math>2u^2 - 1u - 8u + 4</math>  <math>(2u^2 - 1u) - (8u - 4)</math>  <math>u(2u - 1) - 4(2u - 1)</math>  <math>(u - 4)(2u - 1)</math>  <math>(w^2 - 4)(2w^2 - 1) = (w - 2)(w + 2)(2w^2 - 1)</math></p> <p style="text-align: right;"><math>2(4) = 8</math>  <math>\begin{array}{r} -1 \ -8 \\ 2 \ 4 \end{array}</math></p>	<p>6. <math>3x^4 - 5x^2 + 2</math>  <math>3u^2 - 5u + 2</math>  <math>3u^2 - 2u - 3u + 2</math>  <math>(3u^2 - 2u) - (3u - 2)</math>  <math>u(3u - 2) - 1(3u - 2)</math>  <math>(u - 1)(3u - 2)</math>  <math>(x^2 - 1)(3x^2 - 2) = (x + 1)(x - 1)(3x^2 - 2)</math></p> <p style="text-align: right;"><math>3(2) = 6</math>  <math>\begin{array}{r} 1 \ 6 \\ \hline -2 \ -3 \end{array}</math></p>
<p>7. <math>x^4 - 4x^2 + 4</math>  <math>u^2 - 4u + 4</math>  <math>u^2 - 2u - 2u + 4</math>  <math>(u^2 - 2u) - (2u - 4)</math>  <math>u(u - 2) - 2(u - 2)</math>  <math>(u - 2)(u - 2)</math>  <math>(x^2 - 2)(x^2 - 2)</math> OR <math>(x^2 - 2)^2</math></p> <p style="text-align: right;"><math>1(4) = 4</math>  <math>\begin{array}{r} 1 \ 4 \\ \hline -2 \ -2 \end{array}</math></p>	<p>8. <math>y^4 - 6y^2 + 9</math>  <math>u^2 - 6u + 9</math>  <math>u^2 - 3u - 3u + 9</math>  <math>(u^2 - 3u) - (3u - 9)</math>  <math>u(u - 3) - 3(u - 3)</math>  <math>(u - 3)(u - 3) = (y^2 - 3)(y^2 - 3)</math>  OR <math>(y^2 - 3)^2</math></p> <p style="text-align: right;"><math>1(9) = 9</math>  <math>\begin{array}{r} 1 \ 9 \\ \hline -3 \ -3 \end{array}</math></p>
<p>9. <math>3x^4 + 16x^2 - 12</math>  <math>3u^2 + 16u - 12</math>  <math>3u^2 - 2u + 18u - 12</math>  <math>(3u^2 - 2u) + (18u - 12)</math>  <math>u(3u - 2) + 6(3u - 2)</math>  <math>(u + 6)(3u - 2)</math>  <math>(x^2 + 6)(3x^2 - 2)</math></p> <p style="text-align: right;"><math>3(12) = 36</math>  <math>\begin{array}{r} 1 \ 36 \\ \hline -2 \ +18 \\ 3 \ 12 \\ 4 \ 9 \\ 6 \ 6 \end{array}</math></p>	<p>10. <math>2x^4 + 9x^2 - 5</math>  <math>2u^2 + 9u - 5</math>  <math>2u^2 - 1u + 10u - 5</math>  <math>(2u^2 - 1u) + (10u - 5)</math>  <math>u(2u - 1) + 5(2u - 1)</math>  <math>(u + 5)(2u - 1) = (x^2 + 5)(2x^2 - 1)</math></p> <p style="text-align: right;"><math>2(5) = 10</math>  <math>\begin{array}{r} -1 \ +10 \\ 2 \ 5 \end{array}</math></p>

<p>11. <math>2z^4 + 4z^2 - 70</math></p> <p><math>u = z^2</math></p> $\begin{array}{r} 2z^4 + 4z^2 - 70 \\ \underline{2z^4 + 2z^2 - 35} \\ u^2 + 2u - 35 \\ \underline{u^2 - 5u + 7u - 35} \\ (u^2 - 5u) + (7u - 35) \\ u(u-5) + 7(u-5) \\ (u-5)(u+7) \end{array}$ <p><math>1(35) = 35</math></p> $\begin{array}{r} 1 \ 35 \\ \underline{-5 \ 7} \end{array}$ <p><math>2(z^2 - 5)(z^2 + 7)</math></p>	<p>12. <math>3y^4 + 10y^2 - 8</math></p> <p><math>u = y^2</math></p> $\begin{array}{r} 3y^4 + 10y^2 - 8 \\ \underline{3y^4 - 2y^2 + 12y^2 - 8} \\ 3u^2 - 2u + 12u - 8 \\ (3u^2 - 2u) + (12u - 8) \\ u(3u - 2) + 4(3u - 2) \\ (u+4)(3u-2) \end{array}$ <p><math>3(8) = 24</math></p> $\begin{array}{r} 1 \ 24 \\ \underline{-2 \ 12} \\ 3 \ 8 \\ \underline{4 \ 6} \end{array}$ <p><math>(y^2 + 4)(3y^2 - 2)</math></p>
<p>13. <math>4t^4 - 20t^2</math></p> <p><math>u = t^2</math></p> $\begin{array}{r} 4t^4 - 20t^2 \\ \underline{4t^4 - 4t^2} \\ 4t^2 - 5t^2 \\ t^2 - 5 \end{array}$ <p><math>4t^2(t^2 - 5)</math></p>	<p>14. <math>r^4 - 81</math></p> $(r^2 - 9)(r^2 + 9)$ <p><math>(r-3)(r+3)(r^2+9)</math></p>
<p>15. <math>x^4 - x^2 - 12</math></p> <p><math>u = x^2</math></p> $\begin{array}{r} x^4 - x^2 - 12 \\ \underline{x^4 + 3x^2 - 4x^2 - 12} \\ (x^4 + 3x^2) - (4x^2 + 12) \\ u(u+3) - 4(u+3) \\ (u-4)(u+3) \end{array}$ <p><math>1(12) = 12</math></p> $\begin{array}{r} 1 \ 12 \\ \underline{2 \ 6} \\ +3 \ -4 \end{array}$ <p><math>(x^2 - 4)(x^2 + 3) = (x+2)(x-2)(x^2+3)</math></p>	<p>16. <math>w^4 + w^2 - 12</math></p> <p><math>u = w^2</math></p> $\begin{array}{r} w^4 + w^2 - 12 \\ \underline{w^4 - 3w^2 + 4w^2 - 12} \\ (w^4 - 3w^2) + (4w^2 - 12) \\ u(u-3) + 4(u-3) \\ (u+4)(u-3) \end{array}$ <p><math>1(12) = 12</math></p> $\begin{array}{r} 1 \ 12 \\ \underline{2 \ 6} \\ -3 \ 4 \end{array}$ <p><math>(w^2 + 4)(w^2 - 3)</math></p>
<p>17. <math>2y^4 + y^2 - 15</math></p> <p><math>u = y^2</math></p> $\begin{array}{r} 2y^4 + y^2 - 15 \\ \underline{2y^4 - 5y^2 + 6y^2 - 15} \\ (2y^4 - 5y^2) + (6y^2 - 15) \\ u(2u-5) + 3(2u-5) \\ (u+3)(2u-5) \end{array}$ <p><math>2(15) = 30</math></p> $\begin{array}{r} 1 \ 30 \\ \underline{2 \ 15} \\ 3 \ 10 \\ \underline{-5 \ 6} \end{array}$ <p><math>(y^2 + 3)(2y^2 - 5)</math></p>	<p>18. <math>4x^4 - 5x^2 - 9</math></p> <p><math>u = x^2</math></p> $\begin{array}{r} 4x^4 - 5x^2 - 9 \\ \underline{4x^4 + 4x^2 - 9x^2 - 9} \\ (4x^4 + 4x^2) - (9x^2 + 9) \\ 4x^2(x+1) - 9(x+1) \\ (4x-9)(x+1) \end{array}$ <p><math>4(9) = 36</math></p> $\begin{array}{r} 1 \ 36 \\ \underline{2 \ 18} \\ 3 \ 12 \\ \underline{+4 \ -9} \\ 6 \ 6 \end{array}$ <p><math>(4x^2 - 9)(x^2 + 1) = (2x+3)(2x-3)(x^2+1)</math></p>
<p>19. <math>b - 20\sqrt{b} + 64</math></p> <p><math>u = \sqrt{b}</math></p> $\begin{array}{r} b - 20\sqrt{b} + 64 \\ \underline{u^2 - 20u + 64} \\ u^2 - 4u - 16u + 64 \\ (u^2 - 4u) - (16u - 64) \\ u(u-4) - 16(u-4) \\ (u-16)(u-4) \end{array}$ <p><math>1(64) = 64</math></p> $\begin{array}{r} 1 \ 64 \\ \underline{2 \ 32} \\ -4 \ -16 \\ 8 \ 8 \end{array}$ <p><math>(\sqrt{b} - 16)(\sqrt{b} - 4)</math></p>	<p>20. <math>z - 6\sqrt{z} + 8</math></p> <p><math>u = \sqrt{z}</math></p> $\begin{array}{r} z - 6\sqrt{z} + 8 \\ \underline{u^2 - 6u + 8} \\ u^2 - 2u - 4u + 8 \\ (u^2 - 2u) - (4u - 8) \\ u(u-2) - 4(u-2) \\ (u-4)(u-2) \end{array}$ <p><math>1(8) = 8</math></p> $\begin{array}{r} 1 \ 8 \\ \underline{-2 \ -4} \end{array}$ <p><math>(\sqrt{z} - 4)(\sqrt{z} - 2)</math></p>