

Operations with Polynomials
Emphasis on Dividing Polynomials

Divide each of the following using **SYNTHETIC DIVISION**:

$$\begin{array}{r} x-3=0 \\ +3 \quad +3 \\ \hline x=3 \end{array}$$

1. $(x^2 + 7x - 30) \div (x - 3)$

$$\begin{array}{r|rrr} 3 & 1 & 7 & -30 \\ & \downarrow & 3 & 30 \\ \hline & 1 & 10 & 0 \end{array}$$

$x + 10$

2. $(x^2 + 3x - 40) \div (x - 5)$

$$\begin{array}{r|rrr} 5 & 1 & 3 & -40 \\ & \downarrow & 5 & 40 \\ \hline & 1 & 8 & 0 \end{array}$$

$x + 8$

$$\begin{array}{r} x-5=0 \\ +5 \quad +5 \\ \hline x=5 \end{array}$$

$$\begin{array}{r} x-1=0 \\ +1 \quad +1 \\ \hline x=1 \end{array}$$

3. $(x^2 - 13x + 12) \div (x - 1)$

$$\begin{array}{r|rrr} 1 & 1 & -13 & 12 \\ & \downarrow & 1 & -12 \\ \hline & 1 & -12 & 0 \end{array}$$

$x - 12$

4. $(a^2 + 7a - 11) \div (3 - a)$

$$\begin{array}{r|rrr} 3 & 1 & 7 & -11 \\ & \downarrow & 3 & 30 \\ \hline & 1 & 10 & 19 \end{array}$$

$a + 10 + \frac{19}{3-a}$

$$\begin{array}{r} 3-a=0 \\ +a \quad +a \\ \hline 3=a \end{array}$$

$$\begin{array}{r} 1-r=0 \\ +r \quad +r \\ \hline 1=r \end{array}$$

5. $(r^2 + 5r + 7) \div (1 - r)$

$$\begin{array}{r|rrr} 1 & 1 & 5 & 7 \\ & \downarrow & 1 & 6 \\ \hline & 1 & 6 & 13 \end{array}$$

$r + 6 + \frac{13}{1-r}$

6. $(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$

$$\begin{array}{r|rrrr} 4 & 2 & -13 & 26 & -24 \\ & \downarrow & 8 & -20 & 24 \\ \hline & 2 & -5 & 6 & 0 \end{array}$$

$2x^2 - 5x + 6$

$$\begin{array}{r} x-4=0 \\ +4 \quad +4 \\ \hline x=4 \end{array}$$

$$\begin{array}{r} x-2=0 \\ +2 \quad +2 \\ \hline x=2 \end{array}$$

7. $(x^3 - 4x^2 + 6x - 4) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 6 & -4 \\ & \downarrow & 2 & -4 & 4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$x^2 - 2x + 2$

8. $(2x^3 + 3x^2 - 4x + 15) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -4 & 15 \\ & \downarrow & -6 & 9 & -15 \\ \hline & 2 & -3 & 5 & 0 \end{array}$$

$2x^2 - 3x + 5$

$$\begin{array}{r} x+3=0 \\ -3 \quad -3 \\ \hline x=-3 \end{array}$$

$$\begin{array}{r} x-2=0 \\ +2 \quad +2 \\ \hline x=2 \end{array}$$

9. $(3x^3 - 8x^2 + 11x - 14) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & 3 & -8 & 11 & -14 \\ & \downarrow & 6 & -4 & 14 \\ \hline & 3 & -2 & 7 & 0 \end{array}$$

$3x^2 - 2x + 7$

10. $(4a^4 + 2a^2 - 4a + 12) \div (a + 2)$

$$\begin{array}{r|rrrrr} -2 & 4 & 0 & 2 & -4 & 12 \\ & \downarrow & -8 & 16 & -36 & 80 \\ \hline & 4 & -8 & 18 & -40 & 92 \end{array}$$

$4a^3 - 8a^2 + 18a - 40 + \frac{92}{a+2}$

$$\begin{array}{r} a+2=0 \\ -2 \quad -2 \\ \hline a=-2 \end{array}$$

If # in \downarrow is a fraction...

Then mult everthing left of the last # by $\frac{1}{\text{denominator}}$ of the fraction.

$$\begin{array}{r} b-2=0 \\ +2 \quad +2 \\ \hline b=2 \end{array}$$

11. $(6b^4 - 8b^3 + 12b - 14) \div (b - 2)$

$$\begin{array}{r} 2 \overline{) 6 \quad -8 \quad 0 \quad 12 \quad -14} \\ \underline{ 12 \quad 8 \quad 16 \quad 56} \\ 6 \quad 4 \quad 8 \quad 28 \quad +42 \end{array}$$

$$\boxed{6b^3 + 4b^2 + 8b + 28 + \frac{42}{b-2}}$$

12. $(4y^3 - 6y^2 + 4y - 1) \div (2y - 1)$

$$\begin{array}{r} \frac{1}{2} \overline{) 4 \quad -6 \quad 4 \quad -1} \\ \underline{\phantom{\frac{1}{2}} 2 \quad -2 \quad 1} \\ 4 \quad -4 \quad 2 \quad 0 \end{array}$$

$$\frac{1}{2}(4y^2 - 4y + 2) = \boxed{2y^2 - 2y + 1}$$

$$\begin{array}{r} 2y-1=0 \\ +1 \quad +1 \\ \hline 2y=1 \\ \frac{2y}{2} = \frac{1}{2} \\ y = \frac{1}{2} \end{array}$$

$$\begin{array}{r} 3x+1=0 \\ -1 \quad -1 \\ \hline 3x = -1 \\ \frac{3x}{3} = \frac{-1}{3} \\ x = -\frac{1}{3} \end{array}$$

13. $(3x^4 - 5x^3 + x^2 + 7x) \div (3x + 1)$

$$\begin{array}{r} -\frac{1}{3} \overline{) 3 \quad -5 \quad 1 \quad 7 \quad 0} \\ \underline{\phantom{-\frac{1}{3}} -1 \quad 2 \quad -1 \quad -2} \\ 3 \quad -6 \quad 3 \quad 6 \quad -2 \end{array}$$

$$\frac{1}{3}(3x^3 - 6x^2 + 3x + 6) \left(x^3 - 2x^2 + x + 2 - \frac{2}{3x+1} \right)$$

14. $(8x^4 - 4x^2 + x + 4) \div (2x + 1)$

$$\begin{array}{r} -\frac{1}{2} \overline{) 8 \quad 0 \quad -4 \quad 1 \quad 4} \\ \underline{\phantom{-\frac{1}{2}} -4 \quad 2 \quad 1 \quad -1} \\ 8 \quad -4 \quad -2 \quad 2 \quad 3 \end{array}$$

$$\frac{1}{2}(8x^3 - 4x^2 - 2x + 2) \left(4x^3 - 2x^2 - x + 1 + \frac{3}{2x+1} \right)$$

$$\begin{array}{r} 2x+1=0 \\ -1 \quad -1 \\ \hline 2x = -1 \\ \frac{2x}{2} = \frac{-1}{2} \\ x = -\frac{1}{2} \end{array}$$

$$\begin{array}{r} 4y-1=0 \\ +1 \quad +1 \\ \hline 4y = 1 \\ \frac{4y}{4} = \frac{1}{4} \\ y = \frac{1}{4} \end{array}$$

15. $(8y^5 - 2y^4 - 16y^2 + 4) \div (4y - 1)$

$$\begin{array}{r} \frac{1}{4} \overline{) 8 \quad -2 \quad 0 \quad -16 \quad 0 \quad 4} \\ \underline{\phantom{\frac{1}{4}} 2 \quad 0 \quad 0 \quad -4 \quad -1} \\ 8 \quad 0 \quad 0 \quad -16 \quad -4 \quad 3 \end{array}$$

$$\frac{1}{4}(8y^4 - 16y - 4) = \boxed{2y^4 - 4y - 1 + \frac{3}{4y-1}}$$

16. $(15b^3 + 8b^2 - 21b + 6) \div (5b - 4)$

$$\begin{array}{r} \frac{4}{5} \overline{) 15 \quad 8 \quad -21 \quad 6} \\ \underline{\phantom{\frac{4}{5}} 12 \quad 16 \quad -4} \\ 15 \quad 20 \quad -5 \quad 2 \end{array}$$

$$\frac{1}{5}(15b^2 + 20b - 5) = \boxed{3b^2 + 4b - 1 + \frac{2}{5b-4}}$$

$$\begin{array}{r} 5b-4=0 \\ +4 \quad +4 \\ \hline 5b = 4 \\ \frac{5b}{5} = \frac{4}{5} \\ b = \frac{4}{5} \end{array}$$

$$\begin{array}{r} 3c-4=6 \\ +4 \quad +4 \\ \hline 3c = 10 \\ \frac{3c}{3} = \frac{10}{3} \\ c = \frac{10}{3} \end{array}$$

17. $(6c^3 - 17c^2 + 6c + 8) \div (3c - 4)$

$$\begin{array}{r} \frac{4}{3} \overline{) 6 \quad -17 \quad 6 \quad 8} \\ \underline{\phantom{\frac{4}{3}} 8 \quad -12 \quad -8} \\ 6 \quad -9 \quad -6 \quad 0 \end{array}$$

$$\frac{1}{3}(6c^2 - 9c - 6) = \boxed{2c^2 - 3c - 2}$$

18. $(x^2 - 6x - 20) \div (x + 2)$

$$\begin{array}{r} -2 \overline{) 1 \quad -6 \quad -20} \\ \underline{ -2 \quad 16} \\ 1 \quad -8 \quad -4 \end{array}$$

$$\boxed{x - 8 - \frac{4}{x+2}}$$

$$\begin{array}{r} x+2=0 \\ -2 \quad -2 \\ \hline x = -2 \end{array}$$

$$\begin{array}{r} z+3=0 \\ -3 \quad -3 \\ \hline z = -3 \end{array}$$

19. $(3z^4 - 6z^3 - 9z^2 + 3z - 6) \div (z + 3)$

$$\begin{array}{r} -3 \overline{) 3 \quad -6 \quad -9 \quad 3 \quad -6} \\ \underline{ -9 \quad +45 \quad -108 \quad 315} \\ 3 \quad -15 \quad 36 \quad -105 \quad 309 \end{array}$$

$$\boxed{3z^3 - 15z^2 + 36z - 105 + \frac{309}{z+3}}$$

20. $(y^5 - 3y^2 - 20) \div (y - 2)$

$$\begin{array}{r} 2 \overline{) 1 \quad 0 \quad 0 \quad -3 \quad 0 \quad -20} \\ \underline{ 2 \quad 4 \quad 8 \quad 10 \quad 20} \\ 1 \quad 2 \quad 4 \quad 5 \quad 10 \quad 0 \end{array}$$

$$\boxed{y^4 + 2y^3 + 4y^2 + 5y + 10}$$

$$\begin{array}{r} y-2=0 \\ +2 \quad +2 \\ \hline y = 2 \end{array}$$