

**Operations with Polynomials**  
Emphasis on Dividing Polynomials

Divide each of the following using LONG DIVISION:

<p>1. <math>(x^2 + 7x - 30) \div (x - 3)</math></p> $\begin{array}{r} x + 10 \\ x-3 \overline{) x^2 + 7x - 30} \\ \underline{-x^2 + 3x} \phantom{-30} \\ 10x - 30 \\ \underline{-10x + 30} \\ 0 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>x + 10</math></div>	<p>2. <math>(x^2 + 3x - 40) \div (x - 5)</math></p> $\begin{array}{r} x + 8 \\ x-5 \overline{) x^2 + 3x - 40} \\ \underline{-x^2 + 5x} \phantom{-40} \\ 8x - 40 \\ \underline{-8x + 40} \\ 0 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>x + 8</math></div>
<p>3. <math>(x^2 - 13x + 12) \div (x - 1)</math></p> $\begin{array}{r} x - 12 \\ x-1 \overline{) x^2 - 13x + 12} \\ \underline{-x^2 + 1x} \phantom{+12} \\ -12x + 12 \\ \underline{+12x - 12} \\ 0 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>x - 12</math></div>	<p>4. <math>(a^2 + 7a - 11) \div (3 - a)</math></p> $\begin{array}{r} -a - 10 \\ -a+3 \overline{) a^2 + 7a - 11} \\ \underline{-a^2 + 3a} \phantom{-11} \\ 10a - 11 \\ \underline{-10a + 30} \\ 19 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>-a - 10 + \frac{19}{3-a}</math></div>
<p>5. <math>(r^2 + 5r + 7) \div (1 - r)</math></p> $\begin{array}{r} -r - 6 \\ -r+1 \overline{) r^2 + 5r + 7} \\ \underline{-r^2 + r} \phantom{+7} \\ 6r + 7 \\ \underline{-6r + 6} \\ 13 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>-r - 6 + \frac{13}{1-r}</math></div>	<p>6. <math>(2x^3 - 13x^2 + 26x - 24) \div (x - 4)</math></p> $\begin{array}{r} 2x^2 - 5x + 6 \\ x-4 \overline{) 2x^3 - 13x^2 + 26x - 24} \\ \underline{-2x^3 + 8x^2} \phantom{-24} \\ -5x^2 + 26x - 24 \\ \underline{+5x^2 - 20x} \phantom{-24} \\ 6x - 24 \\ \underline{-6x + 24} \\ 0 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>2x^2 - 5x + 6</math></div>
<p>7. <math>(x^3 - 4x^2 + 6x - 4) \div (x - 2)</math></p> $\begin{array}{r} x^2 - 2x + 2 \\ x-2 \overline{) x^3 - 4x^2 + 6x - 4} \\ \underline{-x^3 + 2x^2} \phantom{-4} \\ -2x^2 + 6x - 4 \\ \underline{+2x^2 - 4x} \phantom{-4} \\ 2x - 4 \\ \underline{-2x + 4} \\ 0 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>x^2 - 2x + 2</math></div>	<p>8. <math>(2x^3 + 3x^2 - 4x + 15) \div (x + 3)</math></p> $\begin{array}{r} 2x^2 - 3x + 5 \\ x+3 \overline{) 2x^3 + 3x^2 - 4x + 15} \\ \underline{-2x^3 + 6x^2} \phantom{-4x + 15} \\ -3x^2 - 4x + 15 \\ \underline{+3x^2 + 9x} \phantom{+15} \\ 5x + 15 \\ \underline{-5x - 15} \\ 0 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>2x^2 - 3x + 5</math></div>
<p>9. <math>(3x^3 - 8x^2 + 11x - 14) \div (x - 2)</math></p> $\begin{array}{r} 3x^2 - 2x + 7 \\ x-2 \overline{) 3x^3 - 8x^2 + 11x - 14} \\ \underline{-3x^3 + 6x^2} \phantom{-14} \\ -2x^2 + 11x - 14 \\ \underline{+2x^2 - 4x} \phantom{-14} \\ 7x - 14 \\ \underline{-7x + 14} \\ 0 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>3x^2 - 2x + 7</math></div>	<p>10. <math>(4a^4 + 2a^2 - 4a + 12) \div (a + 2)</math></p> $\begin{array}{r} 4a^3 - 8a^2 + 18a - 40 \\ a+2 \overline{) 4a^4 + 0a^3 + 2a^2 - 4a + 12} \\ \underline{-4a^4 + 8a^3} \phantom{-4a + 12} \\ -8a^3 + 2a^2 - 4a + 12 \\ \underline{+8a^3 + 16a^2} \phantom{-4a + 12} \\ 18a^2 - 4a + 12 \\ \underline{-18a^2 + 36a} \phantom{+12} \\ -40a + 12 \\ \underline{+40a + 80} \\ 92 \end{array}$ <div style="border: 1px solid red; display: inline-block; padding: 5px; margin-left: 200px;"><math>4a^3 - 8a^2 + 18a - 40 + \frac{92}{a+2}</math></div>

11. $(6b^4 - 8b^3 + 12b - 14) \div (b - 2)$	12. $(4y^3 - 6y^2 + 4y - 1) \div (2y - 1)$
13. $(3x^4 - 5x^3 + x^2 + 7x) \div (3x + 1)$	14. $(8x^4 - 4x^2 + x + 4) \div (2x + 1)$
15. $(8y^5 - 2y^4 - 16y^2 + 4) \div (4y - 1)$	16. $(15b^3 + 8b^2 - 21b + 6) \div (5b - 4)$
17. $(6c^3 - 17c^2 + 6c + 8) \div (3c - 4)$	18. $(x^2 - 6x - 20) \div (x + 2)$
19. $(3z^4 - 6z^3 - 9z^2 + 3z - 6) \div (z + 3)$	20. $(y^5 - 3y^2 - 20) \div (y - 2)$

(11)

$$\begin{array}{r}
 6b^3 + 4b^2 + 8b + 28 \\
 b-2 \overline{) 6b^4 - 8b^3 + 12b - 14} \\
 \underline{-6b^4 + 12b^3} \phantom{-14} \\
 4b^3 + 12b - 14 \\
 \underline{-4b^3 + 8b^2} \phantom{-14} \\
 8b^2 + 12b - 14 \\
 \underline{-8b^2 + 16b} \phantom{-14} \\
 28b - 14 \\
 \underline{-28b + 56} \\
 42
 \end{array}$$

$$6b^3 + 4b^2 + 8b + 28 + \frac{42}{b-2}$$

(12)

$$\begin{array}{r}
 2y^2 - 2y + 1 \\
 2y-1 \overline{) 4y^3 - 6y^2 + 4y - 1} \\
 \underline{-4y^3 + 2y^2} \phantom{-1} \\
 -4y^2 + 4y - 1 \\
 \underline{+4y^2 + 2y} \phantom{-1} \\
 2y - 1 \\
 \underline{-2y + 1} \\
 0
 \end{array}$$

$$2y^2 - 2y + 1$$

(13)

$$\begin{array}{r}
 x^3 - 2x^2 + x + 2 \\
 3x+1 \overline{) 3x^4 - 5x^3 + x^2 + 7x} \\
 \underline{-3x^4 + x^3} \phantom{+ 7x} \\
 -6x^3 + x^2 + 7x \\
 \underline{+6x^3 + 2x^2} \phantom{+ 7x} \\
 3x^2 + 7x \\
 \underline{-3x^2 + 1x} \\
 6x
 \end{array}$$

$$x^3 - 2x^2 + x + 2 - \frac{2}{3x+1}$$

$$\begin{array}{r}
 6x \\
 \underline{-6x + 2} \\
 -2
 \end{array}$$

$$\begin{array}{r}
 4x^3 - 2x^2 - x + 1 \\
 \hline
 (14) \quad 2x + 1 \overline{) 8x^4 + 0x^3 - 4x^2 + x + 4} \\
 \underline{-8x^4 + 4x^3} \phantom{+ 4} \\
 -4x^3 - 4x^2 + x + 4 \\
 \underline{+4x^3 + 2x^2} \phantom{+ 4} \\
 -2x^2 + x + 4 \\
 \underline{+2x^2 + x} \\
 \phantom{-} 2x + 4
 \end{array}$$

$$\boxed{4x^3 - 2x^2 - x + 1 + \frac{3}{2x+1}} \quad \frac{2x+4}{-2x+1} \quad 3$$

$$\begin{array}{r}
 2y^4 - 4y - 1 \\
 \hline
 (15) \quad 4y - 1 \overline{) 8y^5 - 2y^4 + 0y^3 - 16y^2 + 0y + 4} \\
 \underline{-8y^5 + 2y^4} \phantom{+ 4} \\
 -16y^2 + 0y + 4 \\
 \underline{+16y^2 + 4y} \phantom{+ 4} \\
 -4y + 4 \\
 \underline{+4y + 1} \\
 \phantom{-} 3
 \end{array}$$

$$\boxed{2y^4 - 4y - 1 + \frac{3}{4y-1}} \quad \frac{-4y+4}{+4y+1} \quad 3$$

$$\begin{array}{r}
 3b^2 + 4b - 1 \\
 \hline
 (16) \quad 5b - 4 \overline{) 15b^3 + 8b^2 - 21b + 6} \\
 \underline{-15b^3 + 12b^2} \phantom{+ 6} \\
 20b^2 - 21b + 6 \\
 \underline{-20b^2 + 16b} \phantom{+ 6} \\
 -5b + 6 \\
 \underline{+5b + 4} \\
 \phantom{-} 2
 \end{array}$$

$$\boxed{3b^2 + 4b - 1 + \frac{2}{5b-4}}$$

$$\begin{array}{r}
 2c^2 - 3c - 2 \\
 3c - 4 \overline{) 6c^3 - 17c^2 + 6c + 8} \\
 \underline{-6c^3 + 8c^2} \phantom{+ 6c + 8} \\
 -9c^2 + 6c + 8 \\
 \underline{+9c^2 + 12c} \phantom{+ 8} \\
 -6c + 8 \\
 \underline{+6c + 8} \\
 0
 \end{array}$$

$$2c^2 - 3c - 2$$

$$\begin{array}{r}
 x - 8 \\
 x + 2 \overline{) x^2 - 6x - 20} \\
 \underline{-x^2 + 2x} \phantom{- 20} \\
 -8x - 20 \\
 \underline{+8x + 16} \\
 -4
 \end{array}$$

$$x - 8 - \frac{4}{x+2}$$

$$\begin{array}{r}
 3z^3 - 15z^2 + 36z - 105 \\
 z + 3 \overline{) 3z^4 - 6z^3 - 9z^2 + 3z - 6} \\
 \underline{-3z^4 + 9z^3} \phantom{- 9z^2 + 3z - 6} \\
 -15z^3 - 9z^2 + 3z - 6 \\
 \underline{+15z^3 + 45z^2} \phantom{+ 3z - 6} \\
 36z^2 + 3z - 6 \\
 \underline{-36z^2 + 108z} \phantom{- 6} \\
 -105z - 6 \\
 \underline{+105z + 315} \\
 309
 \end{array}$$

$$3z^3 - 15z^2 + 36z - 105 + \frac{309}{z+3}$$

$$\begin{array}{r}
 (20) \quad y-2 \overline{) y^4 + 2y^3 + 4y^2 + 5y + 10} \\
 \underline{-y^5 + 2y^4} \phantom{+ 0y^3 + 0y^2 + 0y - 20} \\
 2y^4 - 3y^2 + 0y - 20 \\
 \underline{-2y^4 + 4y^3} \phantom{+ 0y^2 + 0y - 20} \\
 4y^3 - 3y^2 + 0y - 20 \\
 \underline{-4y^3 + 8y^2} \phantom{+ 0y - 20} \\
 5y^2 + 0y - 20 \\
 \underline{-5y^2 + 10y} \phantom{- 20} \\
 10y - 20 \\
 \underline{-10y + 20} \\
 0
 \end{array}$$

$$\boxed{y^4 + 2y^3 + 4y^2 + 5y + 10}$$