Properties of Exponents

Emphasis on Power of a Quotient

Simplify each of the following:

1. $\left(\frac{9d^{-6}}{7d}\right)^2 = \left(\frac{9}{7d^{1}d^{6}}\right)^2 = \left(\frac{9}{7d^{7}}\right)^2$

$= \frac{9^2}{7^2 d^{7\cdot 2}} = \boxed{\frac{81}{49d^{14}}}$

2. $\left(\frac{4b^4}{6b^6w^2}\right)^3 = \left(\frac{4}{6b^6b^{-4}w^2}\right)^3 = \left(\frac{4}{6b^{6+(-4)}w^2}\right)^3$

$= \left(\frac{4^{\div 2}}{6b^2w^2}\right)^3 = \left(\frac{2}{3b^2w^2}\right)^3 = \frac{2^3}{3^3 b^{2\cdot 3}w^{2\cdot 3}}$

$= \boxed{\frac{8}{27b^6w^6}}$ (÷2)

3. $\left(\frac{c^2}{c^{-6}}\right)^3 = (c^2c^6)^3 = (c^{2+6})^3$

$= (c^8)^3$

$= c^{8\cdot 3}$

$= \boxed{c^{24}}$

4. $\left(\frac{9gd^{-4}}{5g^{-2}d^2}\right)^5 = \left(\frac{9g^2g^2}{5d^2d^4}\right)^5 = \left(\frac{9g^{1+2}}{5d^{2+4}}\right)^5$

$= \left(\frac{9g^3}{5d^6}\right)^5 = \frac{9^5 g^{3\cdot 5}}{5^5 d^{6\cdot 5}} = \boxed{\frac{9^5 g^{15}}{5^5 d^{30}}}$

5. $\left(\frac{4w^{-6}}{5w^2}\right)^2 = \left(\frac{4}{5w^2w^6}\right)^2 = \left(\frac{4}{5w^{2+6}}\right)^2$

$= \left(\frac{4}{5w^8}\right)^2 = \frac{4^2}{5^2 w^{8\cdot 2}} = \boxed{\frac{16}{25w^{16}}}$

6. $\left(\frac{d}{d^4}\right)^5 = \left(\frac{1}{d^4d^{-1}}\right)^5 = \left(\frac{1}{d^{4+(-1)}}\right)^5$

$= \left(\frac{1}{d^3}\right)^5$

$= \frac{1^5}{d^{3\cdot 5}} = \boxed{\frac{1}{d^{15}}}$

7. $\left(\frac{b^9}{b^{-2}}\right)^3 = (b^9b^2)^3 = (b^{9+2})^3$

$= (b^{11})^3$

$= \boxed{b^{33}}$

8. $\left(\frac{2}{2^8}\right)^4 = \left(\frac{1}{2^8 2^{-1}}\right)^4 = \left(\frac{1}{2^{8+(-1)}}\right)^4$

$= \left(\frac{1}{2^7}\right)^4 = \frac{1^4}{2^{7\cdot 4}} = \boxed{\frac{1}{2^{28}}}$

9. $\left(\frac{8h^3}{2h}\right)^7 = \left(\frac{8h^3h^{-1}}{2}\right)^7 = \left(\frac{8h^{3+(-1)}}{2}\right)^7$

$= \left(\frac{8h^2}{2}\right)^7 = (4h^2)^7 = 4^7 h^{2\cdot 7}$ (÷2, ÷2)

$= \boxed{4^7 h^{14}}$

10. $\left(\frac{gh}{8g^4h^6}\right)^2 = \left(\frac{1}{8g^4g^{-1}h^6h^{-1}}\right)^2$

$= \left(\frac{1}{8g^{4+(-1)}h^{6+(-1)}}\right)^2$

$= \left(\frac{1}{8g^3h^5}\right)^2 = \frac{1^2}{8^2 g^{3\cdot 2}h^{5\cdot 2}} = \boxed{\frac{1}{64g^6h^{10}}}$

11. $\left(\frac{7^4}{7^{-6}}\right)^5 = (7^4 7^6)^5 = (7^{4+6})^5$

$= (7^{10})^5$

$= 7^{10 \cdot 5}$

$= \boxed{7^{50}}$

12. $\left(\frac{4w^{-6}}{6w^2}\right)^3 = \left(\frac{4}{6w^2 w^6}\right)^3 = \left(\frac{4}{6w^{2+6}}\right)^3$

$= \left(\frac{4^{\div 2}}{6w^8}\right)^3 = \left(\frac{2}{3w^8}\right)^3 = \frac{2^3}{3^3 w^{8 \cdot 3}}$

$\div 2$

$= \boxed{\frac{8}{27w^{24}}}$

13. $\left(\frac{d^2}{d^{-2}}\right)^2 = (d^2 d^2)^2 = (d^{2+2})^2$

$= (d^4)^2$

$= d^{4 \cdot 2}$

$= \boxed{d^8}$

14. $\left(\frac{2c^2}{8c^4}\right)^3 = \left(\frac{2}{8c^4 c^{-2}}\right)^3 = \left(\frac{2}{8c^{4+(-2)}}\right)^3$

$= \left(\frac{2^{\div 2}}{8c^2}\right)^3 = \left(\frac{1}{4c^2}\right)^3 = \frac{1^3}{4^3 c^{2 \cdot 3}}$

$\div 2$

$= \boxed{\frac{1}{64c^6}}$

15. $\left(\frac{gb}{9g^{-2}b^{-5}}\right)^4 = \left(\frac{g^1 g^2 b b^5}{9}\right)^4$

$= \left(\frac{g^{1+2} b^{1+5}}{9}\right)^4 = \left(\frac{g^3 b^6}{9}\right)^4$

$= \frac{g^{3 \cdot 4} b^{6 \cdot 4}}{9^4} = \boxed{\frac{g^{12} b^{24}}{9^4}}$

16. $\left(\frac{6k^8}{2k^4}\right)^2 = \left(\frac{6k^8 k^{-4}}{2}\right)^2 = \left(\frac{6k^{8+(-4)}}{2}\right)^2$

$= \left(\frac{6^{\div 2} k^4}{2_{\div 2}}\right)^2 = \left(\frac{3k^4}{1}\right)^2 = 3^2 k^{4 \cdot 2}$

$= \boxed{9k^8}$

17. $\left(\frac{2^{-2}3^3}{2^{-4}3^6}\right)^6 = \left(\frac{2^{-2} 2^4}{3^6 3^{-3}}\right)^6$

$= \left(\frac{2^{-2+4}}{3^{6+(-3)}}\right)^6 = \left(\frac{2^2}{3^3}\right)^6 = \frac{2^{2 \cdot 6}}{3^{3 \cdot 6}}$

$= \boxed{\frac{2^{12}}{3^{18}}}$

18. $\left(\frac{10a^{23}}{5a^{-15}}\right)^2 = \left(\frac{10a^{23} a^{15}}{5}\right)^2$

$= \left(\frac{10a^{23+15}}{5}\right)^2 = \left(\frac{10^{\div 5} a^{38}}{5_{\div 5}}\right)^2 = (2a^{38})^2$

$= 2^2 a^{38 \cdot 2} = \boxed{4a^{76}}$

19. $\left(\frac{b^3 g^{-8} h^{-5} k^7}{b^{-1} g^2 h^3 k^{-2}}\right)^3 = \left(\frac{b^3 b^1 k^7 k^2}{g^2 g^8 h^3 h^5}\right)^3$

$= \left(\frac{b^{3+1} k^{7+2}}{g^{2+8} h^{3+5}}\right)^3 = \left(\frac{b^4 k^9}{g^{10} h^8}\right)^3$

$= \frac{b^{4 \cdot 3} k^{9 \cdot 3}}{g^{10 \cdot 3} h^{8 \cdot 3}} = \boxed{\frac{b^{12} k^{27}}{g^{30} h^{24}}}$

20. $\left(\frac{3w^{-1}}{9w^6}\right)^4 = \left(\frac{3}{9w^6 w^1}\right)^4 = \left(\frac{3}{9w^{6+1}}\right)^4$

$= \left(\frac{3^{\div 3}}{9^{\div 3} w^7}\right)^4 = \left(\frac{1}{3w^7}\right)^4 = \frac{1^4}{3^4 w^{7 \cdot 4}}$

$= \boxed{\frac{1}{81w^{28}}}$