15 8pts

## Integrated Mathematics Summer School
## Final Pre-Test

**WEEK 1 MATERIAL:**

**Simplify each of the following (2 points each):**

1. $\dfrac{18a^{15}b^{7}}{42ab^{4}}$

$\dfrac{18a^{-1}b^{-4}a^{15}b^{7}}{48}$ (÷6 / ÷6)

$\dfrac{3a^{-1+15}b^{-4+7}}{8} = \boxed{\dfrac{3a^{14}b^{3}}{8}}$

2. $\dfrac{8(3ac)^{4}}{9(3ab)^{2}}$

$= \dfrac{8(3^{1}a^{1}c^{1})^{4}}{9(3^{1}a^{1}b^{1})^{2}}$

$= \dfrac{8\cdot 3^{4}\cdot a^{4}\cdot c^{4}}{9\cdot 3^{2}\cdot a^{2}b^{2}}$

$= \dfrac{8\cdot 3^{4}\cdot 3^{-2}a^{4}\cdot a^{-2}c^{4}}{9b^{2}}$

$= \dfrac{8\cdot 3^{2}a^{4-2}c^{4}}{9b^{2}}$

$= \dfrac{8\cdot 9\cdot a^{2}c^{4}}{9b^{2}} = \boxed{\dfrac{8a^{2}c^{4}}{b^{2}}}$

**Name each of the polynomials by degree and number of terms (2 points each):**

3. $x^4 - 3x^3 + 2x^2 - x + 8$

Highest power in exponent 4 = Quartic

5 terms = Polynomial of 5 terms

So **Quartic Polynomial of 5 terms**

4. $x^5 + 3x - 2$

Highest power in exponent 5 = Quintic

3 terms = Trinomial

So **Quintic Trinomial**

**Simplify each of the following by performing the specified operation (2 points each):**

5. $(7x + 3x^2 - 6) + (3x^2 - 8x + 10)$

$7x + 3x^2 - 6 + 3x^2 - 8x + 10$

$\boxed{6x^2 - x + 4}$

6. $(7x + 3x^2 - 6) - (3x^2 - 8x + 10)$

$7x + 3x^2 - 6 - 3x^2 + 8x - 10$

$\boxed{15x - 16}$

7. $(-2x + 3)(-x + 4)$

$-2x(-x + 4) + 3(-x + 4)$

$2x^2 - 8x - 3x + 12$

$\boxed{2x^2 - 11x + 12}$

8. $(2x^3 + 5x^2 - 32x - 35) \div (x + 1)$

$x + 1 = 0$

$-1 \quad -1$

$x = -1$

| | 2 | 5 | −32 | −35 |
|---|---|---|---|---|
| −1 | | −2 | −3 | 35 |
| | 2 | 3 | −35 | 0 |
| | 2(−1) | 3(−1) | −35(−1) | |

$\boxed{2x^2 + 3x - 35}$

Put in Standard Form!

16 pts

**Completely factor each of the following (2 points each):**

9. $16x^8 - 4x^5$

$\frac{16x^8}{4x^5} \quad \frac{4x^5}{4x^5}$

$4x^5(4x^3 - 1)$

10. $25x^2 - 144$

$\sqrt{25x^2} = 5x \qquad \sqrt{144} = 12$

So

$(5x + 12)(5x - 12)$

$a^2 - b^2 = (a+b)(a-b)$

11. $x^2 + 6x + 5$

$1(5) = 5$

$+1 \; +5$

$x^2 + 1x + 5x + 5$

$(x^2 + 1x) + (5x + 5)$

$x(x+1) + 5(x+1)$

$(x+5)(x+1)$

12. $2x^2 + 3x - 35$

$2(35) = 70$

| | |
|---|---|
| 1 | 70 |
| 2 | 35 |
| 5 | 14 |

$-7 \; +10$

$2x^2 - 7x + 10x - 35$

$(2x^2 - 7x) + (10x - 35)$

$x(2x-7) + 5(2x-7)$

$(x+5)(2x-7)$

13. $x^3 - 216$

$\sqrt[3]{x^3} = x \qquad \sqrt[3]{216} = 6$

$a \qquad\qquad b$

$(x-6)(x^2 + 6x + 36)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

14. $8x^3 + 125$

$\sqrt[3]{8x^3} = 2x \qquad \sqrt[3]{125} = 5$

$a \qquad\qquad b$

$(2x+5)(4x^2 - 10x + 25)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

15. $x^4 - 8x^2 + 7$

Let $u = x^2$

Gives: $u^2 - 8u + 7$

$1(7) = 7$

$-1 \; -7$

$u^2 - u - 7u + 7$

$(u^2 - u) - (7u - 7)$

$u(u-1) - 7(u-1)$

$(u-7)(u-1)$

So $(x^2-7)(x^2-1)$

$x^2 - 1$ see #10

$= (x^2-7)(x+1)(x-1)$

16. $2x^3 + 5x^2 - 32x - 35$

$q = 35 : 1, 5, 7, 35$

$p = 2 : 1, 2$

Try $\pm 1, \pm 5, \pm 7, \pm 35, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{7}{2}, \pm \frac{35}{2}$

| | | | | |
|---|---|---|---|---|
| $-1$ | 2 | 5 | $-32$ | $-35$ |
| | $\downarrow$ | $-2$ | $-3$ | 35 |
| | 2 | 3 | $-35$ | 0 |

$2x^2 + 3x - 35$

see # 12

$(x+5)(2x-7)$

since $x = -1$, $+1 \; +1$

$x + 1 = 0$

$(x+5)(2x-7)(x+1)$

Should have been factor by grouping.



16 pts

**Find the slope of the line containing the given points (2 points each):**

17. $(7,-2)$ & $(-3,4)$

$x_1\ y_1 \quad x_2\ y_2$

$m = \frac{4-(-2)}{-3-7} = \frac{4+2}{-10} = \frac{6}{-10} = \boxed{-\frac{3}{5}}$

18. $(-9,4)$ & $(-11,2)$

$x_1\ y_1 \quad x_2\ y_2$

$m = \frac{2-4}{-11-(-9)} = \frac{-2}{-11+9} = \frac{-2}{-2} = \boxed{1}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

**Write the Linear Equation in Slope-Intercept form using the given information (2 points):**

19. $(-2,13)$ & $(4,17)$

$m = \frac{17-13}{4+2} = \frac{4}{6} = \boxed{\frac{2}{3}}$

$y = \frac{2}{3}x + b$

$13 = \frac{2}{3}\cdot\frac{-2}{1} + b$

$13 = \frac{-4}{3} + b$

$+\frac{4}{3} \quad +\frac{4}{3}$

$\boxed{\frac{43}{3} = b}$

$\boxed{y = \frac{2}{3}x + \frac{43}{3}}$ Answer

20. $(0,-7)$ & $(8,-3)$

$m = \frac{-3+7}{8-0} = \frac{4}{8} = \boxed{\frac{1}{2}}$

$y = \frac{1}{2}x + b$

$-7 = \frac{1}{2}(0) + b$

$-7 = 0 + b$

$\boxed{-7 = b}$

$\boxed{y = \frac{1}{2}x - 7}$ Answer

① Find $m$
② Plug in $m$ & a pt, find $b$.
③ Write the equation.
$y = mx + b$

**WEEK 2 MATERIAL:**

**Solve each of the following Linear Equations (2 points each):**

21. $3x + 12 - 2x = 7x + 2 - 10$

$x + 12 = 7x - 8$

$-x \quad -x$

$12 = 6x - 8$

$+8 \quad +8$

$\frac{20}{6} = \frac{6x}{6}$

$\boxed{x = \frac{10}{3}}$

22. $2(7x+1) - 5 = 25$

$14x + 2 - 5 = 25$

$14x - 3 = 25$

$+3 \quad +3$

$\frac{14x}{14} = \frac{28}{14}$

$\boxed{x = 2}$

**Solve the following Quadratic Equations (2 points each):**

23. $6x^2 - 7x - 5$

$a = 6$

$b = -7$

$c = -5$

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(6)(-5)}}{2(6)}$

$x = \frac{7 \pm \sqrt{169}}{12}$

$x = \frac{7 \pm 13}{12}$

$\frac{7+13}{12} = \frac{20}{12} = \boxed{\frac{5}{3}}$

$\frac{7-13}{12} = \frac{-6}{12} = \boxed{-\frac{1}{2}}$

14 pts

**Solve the given Polynomial (4 points):**

24. $2x^3 + 5x^2 - 32x - 35$

Rational Rt Thm:

$q = 35$ : 1, 5, 7, 35

$p = 2$ : 1, 2

$\pm 1, \pm 5, \pm 7, \pm 35$

$\pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{7}{2}, \pm \frac{35}{2}$

Descartes Rule of signs:

1 change in original
= 1 positive Root

Change all odd exponents

$-2x^3 + 5x^2 + 32x - 35$

2 changes
= 2 or 0 negative Roots.

$-1$ | 2 5 −32 −35
↓ −2 −3 35
2 3 −35 | 0

$-5$ | 2 3 −35
↓ −10 35
2 −7 | 0

$2x - 7 = 0$
$+7 \quad +7$
$\frac{2x}{2} = \frac{7}{2}$

So $x = -1, -5, \frac{7}{2}$

**Solve the following Exponential Equations (2 points each):**

25. $3^{5x+8} = 81^{-2x+3}$

$3^{5x+8} = (3^4)^{-2x+3}$

So $5x + 8 = 4(-2x + 3)$
$5x + 8 = -8x + 12$
$+8x \quad -8 \quad +8x \quad -8$
$\frac{13x}{13} = \frac{4}{13}$

$x = \frac{4}{13}$

**Solve the following Logarithmic Equations (2 points each):**

26. $\log_2(x + 3) - \log_2(3x + 1) = 6$

$\log_2 \frac{x+3}{3x+1} = 6 \rightarrow \frac{x+3}{3x+1} = 2^6 \rightarrow \frac{x+3}{3x+1} = \frac{64}{1} \rightarrow$

$1(x+3) = 64(3x+1)$
$x + 3 = 192x + 64$
$-x \quad -64 \quad -x \quad -64$
$\frac{-61}{191} = \frac{191x}{191}$

$\log_b mn = \log_b m + \log_b n$

$\log_b \frac{m}{n} = \log_b m - \log_b n$

$\log_b y = x \Leftrightarrow y = b^x$

**Identify the midpoint of each of the following (2 points each):**

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

27. $(-10, 15)$ & $(20, 7)$

$\left(\frac{-10+20}{2}, \frac{15+7}{2}\right) = \left(\frac{10}{2}, \frac{22}{2}\right) = (5, 11)$

28. $(8, -7)$ & $(-4, 11)$

$\left(\frac{8-4}{2}, \frac{-7+11}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2)$

**Find the distance between the given points:**

$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

29. $(-10, 15)$ & $(20, 7)$

$\sqrt{(20+10)^2 + (7-15)^2}$

$\sqrt{(30)^2 + (-8)^2}$

$\sqrt{900 + 64} = \sqrt{964}$

30. $(8, -7)$ & $(-4, 11)$

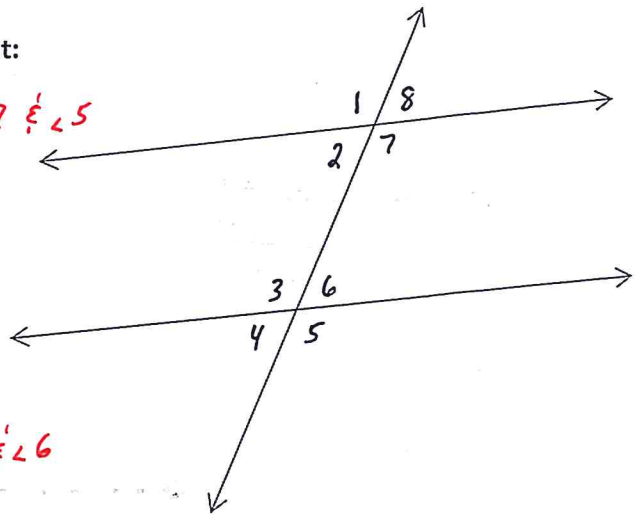
$\sqrt{(-4-8)^2 + (11+7)^2}$

$\sqrt{(-12)^2 + (18)^2}$

$\sqrt{144 + 324} = \sqrt{468}$

16 pts

**Identify each of the following using the diagram to the right:**




31. List all the pairs of Corresponding Angles.

∠1 ≅ ∠3, ∠2 ≅ ∠4, ∠8 ≅ ∠6, ∠7 ≅ ∠5

32. List all the pairs of Alternate Interior Angles.

∠2 ≅ ∠6 ∠3 ≅ ∠7

33. List all the pairs of Alternate Exterior Angles.

∠1 ≅ ∠5 ∠4 ≅ ∠8

34. List the Same-Side Interior Angles.

∠2 ≅ ∠3 ∠6 ≅ ∠7

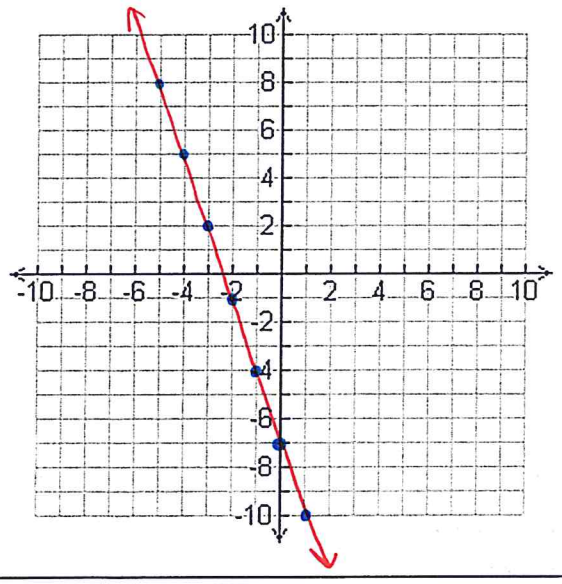
35. List all of the Vertical Angles.

∠1 ≅ ∠7, ∠2 ≅ ∠8 ∠3 ≅ ∠5 ∠4 ≅ ∠6

**Graph each of the following functions:**

36. LINEAR

$y = -3x - 7$

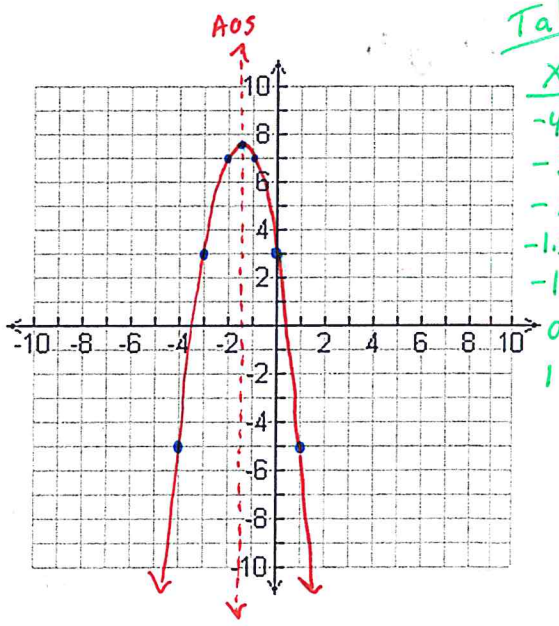


$m = \frac{-3}{1}$ down 3 right 1 Reverse: up 3 left 1

$b = -7$

37. QUADRATIC

$y = -2x^2 - 6x + 3$




Table

| x | y |
|---|---|
| -4 | -5 |
| -3 | 3 |
| -2 | 7 |
| -1.5 | 7.5 |
| -1 | 7 |
| 0 | 3 |
| 1 | -5 |

AOS

$x = \frac{-b}{2a} = \frac{6}{2(-2)} = \frac{6}{-4} = \frac{-3}{2}$

$a = -2$

$b = -6$

$c = 3$



34 pts

**Graph each of the following functions:**

38. RATIONAL FUNCTION

$$f(x) = \frac{x^2 + 8x + 12}{x^2 - 5x - 14} = \frac{(x+2)(x+6)}{(x+2)(x-7)}$$

Hole at $x+2=0$, $-2 \quad -2$, $x = -2$

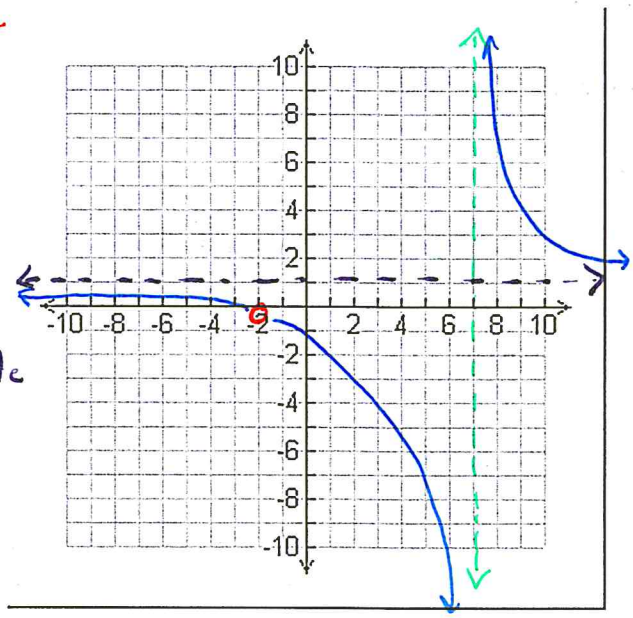
Vertical Asymptote

$x - 7 = 0$, $+7 \quad +7$, $x = 7$

Horizontal Asymptote

$y = \frac{x^2}{x^2} = 1$

so $y = 1$

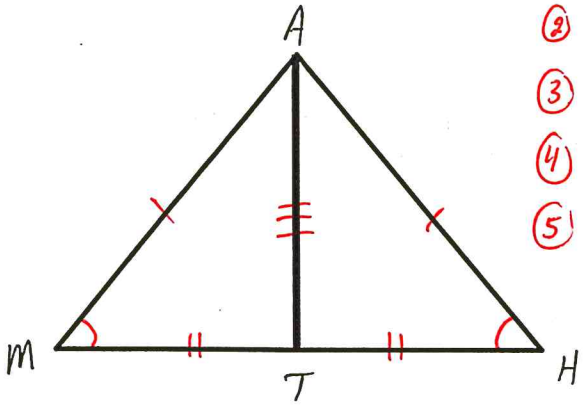



**WEEK 3 MATERIAL:**

**Prove the following triangles are CONGRUENT using SSS, SAS, ASA, AAS or HL:**

39. **Given:** Δ MAH is an equilateral Δ
T is the midpoint of $\overline{MH}$

**Prove:** Δ MTA ≅ Δ HTA




| Statements | Reasons |
|---|---|
| ① Δ MAH is an equilateral Δ, T midpt of $\overline{MH}$ | ① Given |
| ② $\overline{MA} \cong \overline{HA}$ ; $\angle M \cong \angle H$ | ② Definition Equilateral |
| ③ $\overline{MT} \cong \overline{HT}$ | ③ Definition Midpoint |
| ④ $\overline{AT} \cong \overline{AT}$ | ④ Reflexive Property |
| ⑤ Δ MTA ≅ Δ HTA | ⑤ SSS or SAS |

16 pts

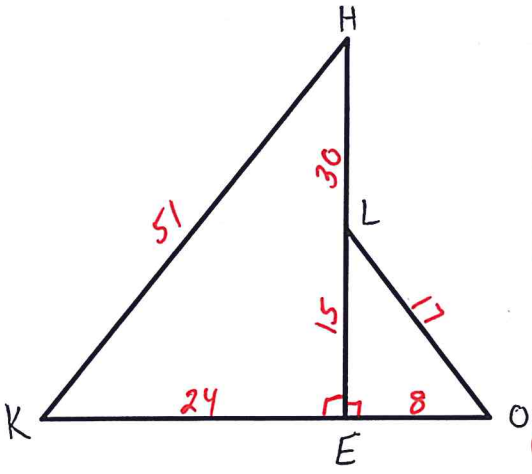
**Prove the following triangles are SIMILAR using SSS, SAS, or AA:**

40. **Given:** $\overline{HE} \perp \overline{KO}$

KE = 24, EO = 8, LO = 17

HL = 30, LE = 15, KH = 51

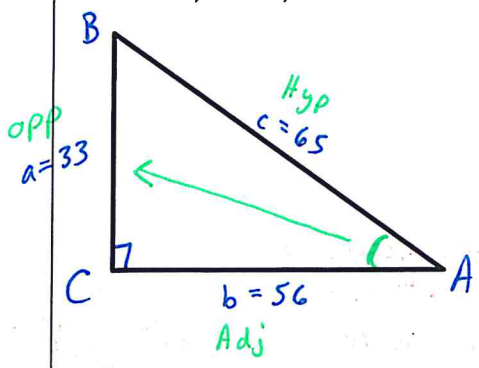
Prove: Δ KHE ~ Δ OLE




| Statements | Reasons |
|---|---|
| ① $\overline{HE} \perp \overline{KO}$, KE = 24, EO = 8, LO = 17, HL = 30, LE = 15, KH = 51 | ① Given |
| ② ∠KEH & ∠OEL are Rt ∠'s | ② Definition ⊥ |
| ③ ∠KEH ≅ ∠OEL | ③ Right Angles Congruence Theorem. |
| ④ $\frac{KH}{OL} = \frac{51}{17} = \frac{3}{1}$ | ④ ÷ T/B by 17 |
| ⑤ $\frac{HE}{LE} = \frac{45}{15} = \frac{3}{1}$ | ⑤ ÷ T/B by 15 |
| ⑥ $\frac{KE}{OE} = \frac{24}{8} = \frac{3}{1}$ | ⑥ ÷ T/B by 8 |
| ⑦ $\frac{KH}{OL} = \frac{HE}{LE} = \frac{KE}{OE} = \frac{3}{1}$ | ⑦ Transitive Property |
| ⑧ Δ KHE ~ Δ OLE | ⑧ SSS or SAS |

**Evaluate the six trigonometric functions for the regular right triangle using ∠ A:**

41. a = 33, b = 56, & c = 65




SOH - CAH - TOA

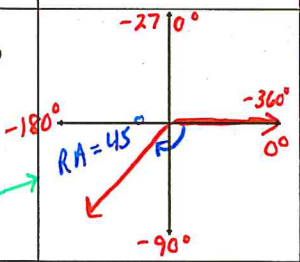
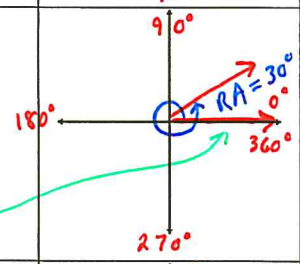
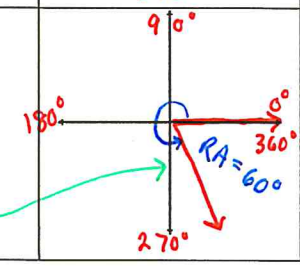
$\sin A = \frac{33}{65}$ $\quad \csc A = \frac{65}{33}$

$\cos A = \frac{56}{65}$ $\quad \sec A = \frac{65}{56}$

$\tan A = \frac{33}{56}$ $\quad \cot A = \frac{56}{33}$

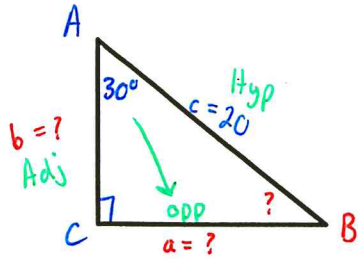
12 pts

**Using the hand trick discussed in class, give the exact value of each of the following trig functions:**

| Given Angle | Reference Angle | Quadrant Terminal Side is located in | Exact Value |
|---|---|---|---|
| 42. $\sin -135^o$ <br> 180 <br> -135 |   | Quadrant III <br> Only Tan is positive in III | $-\frac{\sqrt{2}}{2}$ |
| 43. $\cos 390^o$ <br> 390 <br> -360 |   | Quadrant I <br> All Functions are positive | $\frac{\sqrt{3}}{2}$ |
| 44. $\tan 300^o$ <br> 360 <br> -300 |   | Quadrant IV <br> Only cos is positive in IV | $-\sqrt{3}$ |

**Solve the following triangles given the information provided on a separate sheet of paper:**

45. $\angle A = 30^o$ & $c = 20$




$\angle B = 180 - 90 - 30$

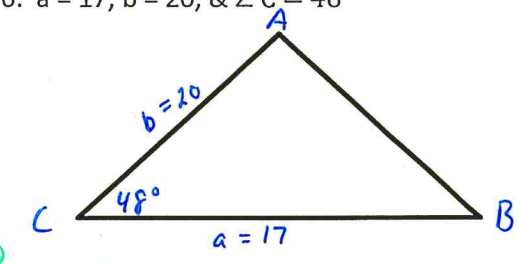
$\angle B = 60°$

$20 \cdot \sin 30 = \frac{a}{20} \cdot 20$ $\quad 20 \cdot \cos 30 = \frac{b}{20} \cdot 20$

$20 \sin 30 = a$ $\quad 20 \cos 30 = b$

$10 = a$ $\quad b \approx 17.3$

46. $a = 17$, $b = 20$, & $\angle C = 48^o$




① $c^2 = 17^2 + 20^2 - 2(17)(20)\cos 48$

$c^2 = 289 + 400 - 680 \cos 48$

$\sqrt{c^2} = \sqrt{689 - 680 \cos 48}$

$c \approx 15.296\,7705$

$c \approx 15.3$

② $\frac{\sin A}{17} = \frac{\sin B}{20} = \frac{\sin 48}{15.3}$

$\frac{15.3 \sin B}{15.3} = \frac{20 \sin 48}{15.3}$

$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{20 \sin 48}{15.3}\right)$

$\angle B \approx 76.2714885$

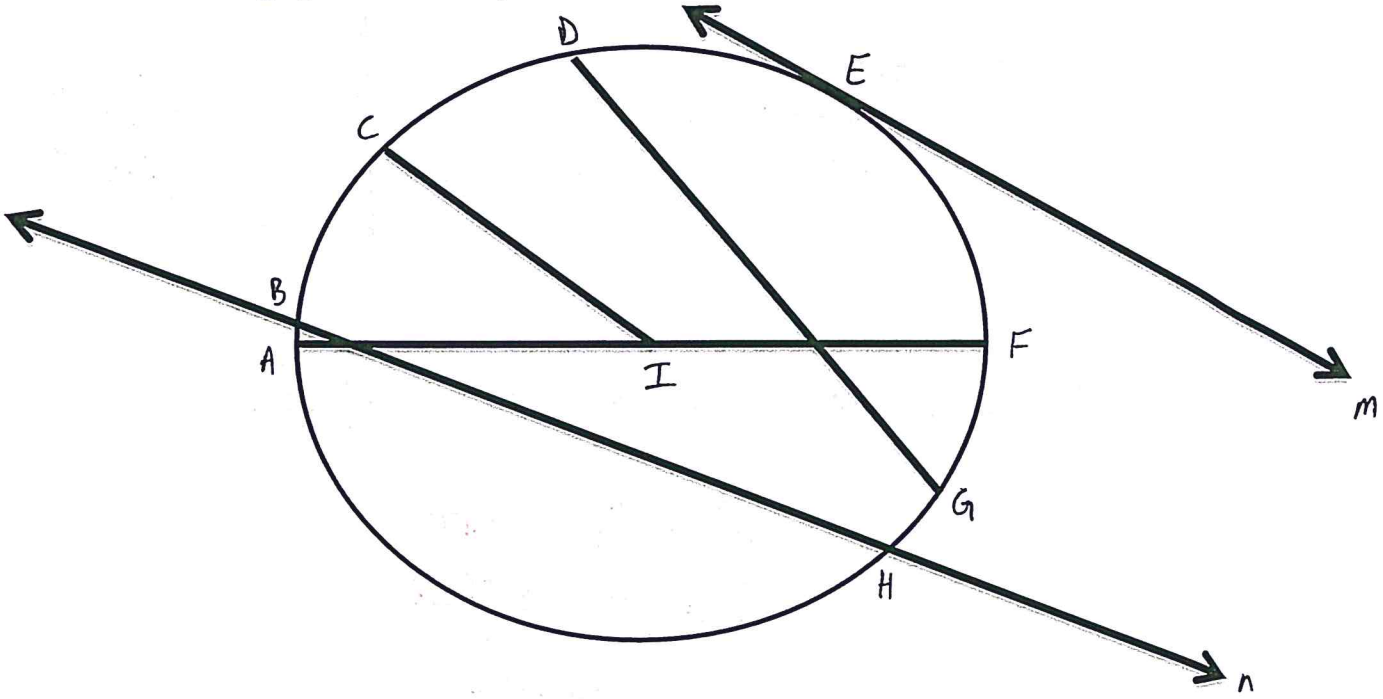
$\angle B \approx 76.3°$

③ $\angle A = 180 - 48 - 76.3$

$\angle A \approx 55.7°$

15 pts

**Use the following figure to answer questions #47 - #53:**



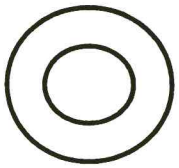

| | |
|---|---|
| 47. Name any chords shown above.<br>$\overline{AF}$ & $\overline{DG}$ & $\overline{BH}$ | 48. Name the secant shown above.<br>$\overleftrightarrow{BH}$ or line n |
| 49. Name the diameter shown above.<br>$\overline{AF}$ | 50. Name all radi shown above.<br>$\overline{AI}$, $\overline{CI}$, & $\overline{FI}$ |
| 51. Name the tangent shown above.<br>line m | 52. Name the point of tangency shown above.<br>E |
| 53. Name the circle shown above.<br>$\odot I$ | |



11 pts

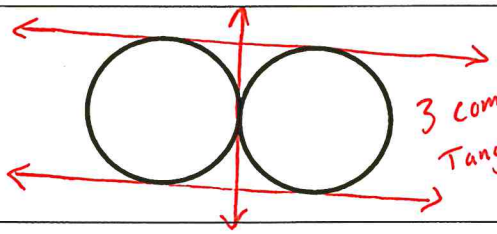
**Identify the number of and draw each common tangent in each of the following figures:**

54.



Zero Common Tangents

55.



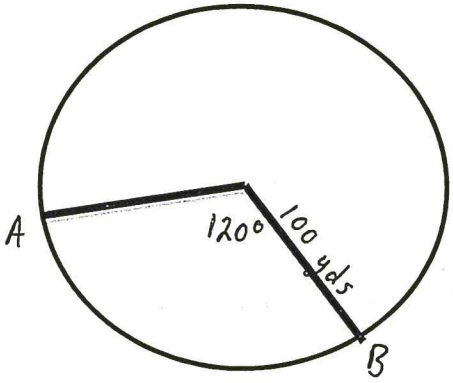
3 common Tangents

**Find the indicated measure:**

56. Find the circumference of a circle with a diameter of 100 yards.

$C = \pi d \rightarrow C = \pi(100) \rightarrow C = \boxed{100\pi}$

57. Find the length of $\overset{\frown}{AB}$ given




$\overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360} \cdot 2\pi r$

$\overset{\frown}{AB} = \frac{120}{360} \cdot \frac{2\pi(100)}{1}$

$\overset{\frown}{AB} = \frac{120 \cdot 2 \cdot 100\pi}{360 \cdot 1}$

$\overset{\frown}{AB} = \frac{24000\pi}{360}$

$\overset{\frown}{AB} = \boxed{\frac{200}{3}\pi}$

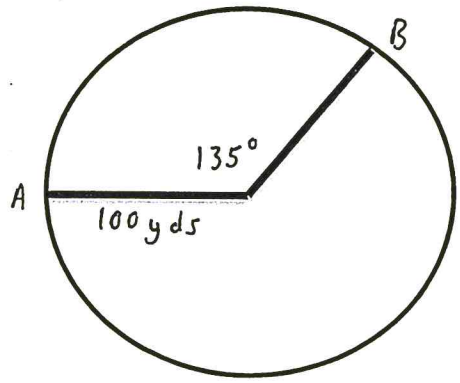
**Find the indicated measure:**

58. Find the area of a circle with a diameter of 100 yards.

$d = 2r$, $\frac{100}{2} = \frac{2r}{2}$, $r = 50$

$A = \pi r^2 \rightarrow A = \pi(50)^2 \rightarrow A = \boxed{2500\pi}$

59. Find the area of the sector given:




$A_{sec} = \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2$

$A_{sec} = \frac{135}{360} \cdot \frac{\pi(100)^2}{1}$

$A_{sec} = \frac{135(100)^2\pi}{360 \cdot 1}$

$A_{sec} = \frac{1350000\pi}{360}$

$A_{sec} = \boxed{3750\pi}$

8 pts